Chapter 3

On the Plurality of Quantum Metaphysics

In the previous chapter we briefly presented Bohmian mechanics, GRW, and the many worlds theory. Each of them can be considered to be an answer to the problem of constructing a quantum theory without observer, and each of themy can be considered to be a reaction to what Bell considered the obvious moral of the problem of the cat: either Schrödinger's evolution is not always correct or the state of the system is not described only by the wave function. Then we have the many worlds theory, that corresponds, as we saw, to the attempt of making sense of the idea that all measurement results are realized.

In this and in the following chapters I will discuss these theories from two different perspectives and discuss what are the possible metaphysical morals that we can get from these theories.

3.1 Bohmian Mechanics

As we discussed in the previous chapter, in Bohmian mechanics the state of a physical system at a given time t is given by the couple (Ψ_t, Q_t). The wave function Ψ evolves according to Schrödinger equation, and the particles evolve according to the guide equation determined by the wave function, see equations (2.5) and (2.6).

Now, let us try to infer something about metaphysics from the hypothesis that this theory is true. What is there if Bohmian mechanics is true? We have already answered to this question: particles and the wave function constitute the state, which provides the ontology of the theory. But if one instead asks the question: "What are tables and chairs made of if Bohmian mechanics is true?", then we can provide different answers. In fact, it is not obvious that all there is gets to constitute physical objects. Only if we assume so, the two questions coincide. But if, for example, we believe in the existence of numbers, of laws of nature, of consciousness, then of course they are part of the ontology even if they do not constitute any physical object. So let us now to analyze what are the possible alternative answers to the question: What is the primitive ontology of Bohmian mechanics?

3.1.1 Particle Bohmian Mechanics

The most natural, at least to me, interpretation of Bohmian mechanics is the one provided by Bohm and Hiley (Bohm and Hiley 1993), by Bell (Bell 1987) and by Dürr, Goldstein, Tumulka, Zanghí and myself (DGZ 1992), (DGZ 1997), (DGZ 2004) (Allori et al. 2007): it is a theory of particles in motion. That is, tables and chairs are made of particles: they are the primitive ontology of the theory. In the usual terminology, the actual positions $Q_1, ..., Q_N$ in $\mathbb{R}^3$ of the particles are considered the *hidden variables* of the theory: the variables which, together with the wave function, provide a complete description of the system, the wave function alone providing only a partial, incomplete, description. This terminology suggests that the "main" object of the theory is the wave function, with the additional information provided by the particles' positions, which seem to play in some respect some sort of secondary role: we added them, it seems, just to "recover" the properties of the macroscopic objects. The situation is instead very much the opposite: if Bohmian mechanics is a theory of particles, their positions are the *primitive* variables, and the description in terms of them must be completed by specifying the wave function to define their dynamics. In other words, according to this view, Bohmian mechanics is a theory is about particles in three-dimensional space, while the wave function is *not* to be considered as a material field. In this regard, Dürr, Goldstein and Zanghí write:

> if one focuses directly on the question as to what the theory is about, one is naturally led to the view that Bohmian mechanics is fundamentally about the behavior of particles, described by their positions - or fields, described by field configurations, or strings, described by string configurations - and only secondarily about the behavior of wave functions (DGZ 1997).

A similar attitude is the one of David Bohm and Basil Hiley in their book *The Undivided Universe*:

> In our interpretation, however, we do not assume that the basic reality is thus described primarily by the wavefunction. Rather [...] we begin with the assumption that there are particles following definite trajectories [...] We then assume that the wavefunction, describes a qualitatively new kind of quantum field which determines the guidance conditions and the quantum potential acting on the particle. We are not denying the reality of this field, but we are saying that its significance is relatively subtle in the sense that it contains active information that 'guides the particle in its self-movement under its own energy [...] So ultimately all manifestations of the quantum fields are through the particles (Bohm and Hiley 1993).

Therefore, in Bohmian mechanics particles are the primitive ontology of the theory. Particles constitute what tables and chairs are made of, they are the little lego bricks that build up every physical object. We will call this approach to Bohmian mechanics, "particle Bohmian mechanics" or BMp, to differentiate to the approach that we will see in the next section.

If Bohmian mechanics is a theory of particles, what is the wave function in this view? There can be more than one choice, as we will discuss further in Section 4.6. According to Dürr, Goldstein and Zanghí and to Bohm and Hiley, the wave function is part of the furniture of the world, it is real. But, contrarily to what happens in the case of the primitive ontology, which constitutes matter, it does not constitute matter but, rather, it tells matter how to move. It provides the law for the motion of the particles, just as the potential in classical mechanics is the generator of the motion of the positions of the particles. Dürr, Goldstein and Zanghí [1] have a Platonic view of law, then we have no problem to regard the wave function as real insofar as laws are real. But there is nothing that forces us to do so, especially if one has a Humean view of laws. In that case, one might be less committed to the existence of the wave function, since it would just turn out to be our best and most informative way of writing down the theory of the world. Or if one is a Nominalist with respect to laws, one would naturally regard the wave function as nonexisting.

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[1] Private communication.

3.1.2 Configuration Bohmian Mechanics

In the previous section we discussed Bohmian mechanics as a theory of particles, in which the wave function was not something that constitutes matter. In contrast, in this section I will analyze Bohmian mechanics as a theory according to which tables and chairs are indeed made of particles but also of wave functions.

From the fact that in the theory there is the wave function, it might be natural to conclude that it has to be describing a physical object. Not just real, but real *and* physical. And if we start taking the wave function so seriously as a physical entity, then we should indeed realize that the space in which it lives should be real too. So we end up having, in this view, both three-dimensional space, where the particles live, and configuration space, where the wave function lives. One might say that both of these spaces are indeed part of physical space. This is what Bradley Monton in his *Wave Function Ontology* (Monton 2002) calls the "mixed ontology":

> A proponent of a mixed ontology would hold that, in addition to the 3N-dimensional space, there also exists a three-dimensional space.

Dürr, Goldstein and Zanghí would opt to say instead that physical space is $\mathbb{R}^3$, while $\mathbb{R}^{3N}$ is just a space constructed from physical space for mathematical convenience. Another, maybe more radical move is to say exactly the opposite: physical space *is* configuration space, while three-dimensional space is just an illusion. This is David Albert's idea, that he expresses in *Elementary Quantum Metaphysics* (Albert 1996). He notes that, given that the wave function is an object in $\mathbb{R}^M$, with $M = 3N$, it is physical space in the case of Bohmian mechanics, as well as in any theory in which the wave function is taken to represent physical objects. As a consequence, $Q \in \mathbb{R}^{3N}$ is actually not describing N particles in three dimensional space $\mathbb{R}^3$. That is, Q is not the n-tuple $(Q_1, ..., Q_N)$, where Q_k is a triplet in $\mathbb{R}^3$. Rather, the whole universe is just a single point Q in the highly dimensional $\mathbb{R}^M$ together with the wave function Ψ, that should be intended as well as a concrete, physical field in such a space. In Albert's words:

> it has been essential [...] to the project of quantum-mechanical *realism* (in *whatever* particular form it takes - Bohm's theory, or modal theories, or Everettish

> theories or theories of spontaneous localization), to learn to think of the wave functions as physical objects *in and of themselves.* And of course the space those sorts of objects *live* in, and (therefore) the space *we* live in, the space in which any realistic understanding of quantum mechanics is necessarily going to depict the history of the world as *playing itself out* (if space is the right name for it - of which more later) is *configuration space.* And whatever impression we have of the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional space-time) is somehow flatly illusory. [...] In Bohm's theory, the world will consist of exactly two physical objects. One of those is the universal wave function and the other is the universal *particle* (Albert 1996).

In contrast with the view presented earlier, here configuration space plays a very fundamental role: it is the fundamental physical space in which we live. For this reason we might call this theory configuration Bohmian mechanics or cBM, as opposed to BMp presented above. The name "configuration" is misleading, since this space is primitive, it is not constructed from the configuration of particles in three-dimensional space. In fact there are no three-dimensional particles, and there is no three-dimensional space.

Now, it seems clear that any complete, fundamental physical theory must account for the behavior of macroscopic objects of three-dimensional space. In other words, we could say that what physics ought to be able to do is to account for what appears to be happening around us. If we take configuration space to be physical space, then we need to explain why it seems as if we live in a three-dimensional space. Given that the wave function is in configuration space and it is all there is, and given that this space is primitive, we do not have enough resources to recover or deduce three-dimensional space without making use of the very definition of configuration. In fact, if the theory talks about the behavior of one single point Q and a field Ψ in this highly dimensional space (call this dimension M and forget that $M = 3N$), then we have no more that that. We might be tempted to regard the coordinates of Q as grouped into triples Q_k, such that $Q = (Q_1, ..., Q_N)$, representing the three spatial coordinates of the "particles". And, accordingly, to consider the variables of the wave function as $\Psi(Q) = \Psi(Q_1, ..., Q_N)$, where a single coordinate Q_k is a triplet of terms, each of them is in $\mathbb{R}^3$ and represents a spatial coordinate of a "particle". But we simply do not have any justification to do that: there are no three-dimensional particles in this theory, just one single particle in configuration space and a field in that space. The only way we could make the identification of Q with $(Q_1, ..., Q_N)$, $Q_k \in \mathbb{R}^3$, is to *already* know that the configuration can be divided as such. That is, we can interpret the word "configuration" as the N-tuple of the positions of N three-dimensional particles if we assume there are N particles in $\mathbb{R}^3$. As Monton also emphasized (Monton 2002), one could think that $\mathbb{R}^3$ supervenes on $\mathbb{R}^{3N}$ so that one does not really need to postulate the existence of a three-dimensional space. But that it is simply false, since there are many ways in which N particles could evolve in three-dimensional space and give rise to the same configuration space. The point is that we simply have not enough structure to read off only from configuration space the three-dimensional world. In short, there is an explanatory gap between the behavior of something on a highly dimensional abstract space and the behavior of objects in three-dimensional space. Therefore, if one wants the theory to be a satisfactory fundamental physical theory, one has to add to the specification of the theory some rule in order to recover three-dimensional space from it.

Albert agrees that we need some map from $\mathbb{R}^M$ to $\mathbb{R}^3$, and in particular he claims that it is the Hamiltonian that gives us such a rule (Albert 1996). His reasoning goes as follows. Physical space is $\mathbb{R}^M$, where it happens to be the case that $M = 3N$, even if this does not mean that there is some *a priori* reason to group the components of Q in triples $Q_k \in \mathbb{R}^3$, as we have seen before. The total Hamiltonian of the world is something like

$$H = \nabla_q^2 + V(q), \tag{3.1}$$

where $q \in \mathbb{R}^M$. Without any further restrictions, this Hamiltonian could apply to a space in which we have different groupings of the coordinates of q. But, Albert claims, it is an *empirical fact* of the world that the potentials V should be written as

$$V(q) = \sum_{i<j} V(|q_i - q_j|),$$

where $q = (q_1, ..., q_N)$, $q_k \in \mathbb{R}^3$, for any $k = 1, ..., N$. And this is what insures us of the appearances of the world as three-dimensional. The structure of the actual Hamiltonian, Albert says, is the one that picks as natural the grouping in terms of triplets and therefore explains why we think we live in a three-dimensional space. Note that there is no further explanation of why the Hamiltonian is the way it is or the dimensionality M of physical space is what it is: for example, there is no explanation of why M cannot be a prime number [2].

In addition, a theory should be able to account of the assignment of properties to macroscopic objects, as we do in terms of our ordinary language. For example, it will need to explain what do we mean when we say: "the table is localized in the middle of the room" in the terms of the fundamental physical theory. What does it mean to say that there is a table in the middle of the room, in cBM? As we have anticipated at the end of the previous chapter, in order to answer this question we need to add a rule of correspondence between the macroscopic language and the microscopic one.

3.2 GRW Theory

Spontaneous collapse theories are usually characterized a theories in which the wave function provides the complete description of the system but does not evolve according to Schrödinger's equation [3]. It evolves according to a stochastic nonlinear equation that makes it the case that the linear evolution of Schrödinger equation is interrupted randomly by collapses. As we did for Bohmian mechanics, let us ask ourselves: What are tables and chairs made of if GRW is true? What is the primitive ontology of GRW theory?

3.2.1 Bare GRW

As it was somewhat natural to regard Bohmian mechanics as a theory of particles, it seems natural to regard GRW as a theory about the wave function. After all, is not it the object that appear in the modified Schrödinger's equation? This is the view, entertained for example by people like David Albert in the previosly mentioned

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[2] This has been suggested by Tim Maudlin, private communication.

[3] As anticipated, in Chapter 4 I will challenge this characterization.

(Albert 1996), Peter Lewis in his *Life in Configuration Space* (P. Lewis 2004) and Alberto Rimini and Oreste Nicrosini in their *Relativistic Spontaneous Localization: a Proposal* (Rimini and Nicrosini 2003). This position, that we will call GRW∅ to emphasize its minimality, could be summarized as follows: the wave function is what GRW theory is about, the wave function is a real, physical field,

> just like electromagnetic fields in classical electrodynamics (Albert 1996).

With the difference that the wave function now lives in a much bigger space: configuration space, the space of all the positions of all the particles in the universe.

Therefore, since in GRW∅ all there is is the wave function, and it lives in configuration space, physical space is configuration space, as in cBM. Also as in cBM, we have the problem of explaining how it is the case that, even if physical space has a dimension much bigger than 3, it looks as if it were three-dimensional. As emphasized in Section 3.1.2, there is an explanatory gap between the behavior of something on a highly dimensional abstract space and the behavior of objects in three-dimensional space so that it is necessary to add some rule to GRW∅ in order to recover the three-dimensional space of our experience. As in cBM, it is the Hamiltonian that provides the connection.

As underlined in Section 2.5, in GRW∅ we have the problem of the tails, the problem of assigning properties to macroscopic physical objects given their microscopic description. As we have mentioned earlier, we might run into trouble, as Albert and Loewer point out in (Albert and Loewer 1996), if we assume that the usual eigenstate-eigenvalue link of orthodox quantum mechanics holds also in GRW∅. In fact this, as already discussed in the second chapter, would cause certain properties, like the localization of a macroscopic object for example, which we think are defined to remain undefined instead. Therefore, we need to have a different rule to connect ordinary macroscopic appearances with what the physics tells us in terms of wave functions. Albert and Loewer have proposed the fuzzy link. In this way, they say, it is possible to recover what we usually mean when we talk about localizable objects on the macroscopic level and the appearances of those objects to be localized while they are not.

3.2.2 Flashy GRW

What we discussed in 2.4.2 is, more or less, all there is to say about the mathematical formulation of the GRW theory according to most people. The linear evolution of the wave function is interrupted by collapses. When the wave function is ψ, a collapse of center x and label k occurs at rate given by equation (2.11), and when this happens, the wave function changes to $\mathsf{L}_i(x)^{1/2}\psi/\|\mathsf{L}_i(x)^{1/2}\psi\|$. Contrarily to the idea that it is natural to consider GRW as a theory about the wave function Gian Carlo Ghirardi, the "G" in GRW and the leading supporter of the theory, and Bell believe that the description in terms of the wave function is not the whole story. Concerning this possibility, Bell noted the following:

> But the wave function as a whole lives in a much bigger space, of $3-N$ dimensions. It makes no sense to ask for the amplitude or phase or whatever of the wave function at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary three-space are specified (Bell 1987).

Therefore, the wave function alone does not seem to be able to describe physical objects. He then adds:

> However GRW jumps (which are part of the wave function, not something else) are well localized in ordinary space. Indeed each is centered on a particular space time point (x, t). So we can propose these events as the basis of the"local beable" of the theory (Bell 1987).

The idea is that GRW can account for the structure of events in three-dimensional space in terms of the points $(X_k, T_k), k = 1, ..., N$ of space-time that correspond to the localization events of the wave function. Once a history of the wave function is given, from an initial time $t = 0$ to a given time t, also the set of points in space-time, representing the localization events happened between the initial time an the time t,

$$F = \{(X_1, T_1), (X_2, T_2), \dots (X_i, T_i), \dots (X_K, T_K), \dots\}, \tag{3.2}$$

with $T_1 < T_2 < \dots < T_k < \dots$, is determined. Bell's proposal is to consider F as the image of the world that is given us by the GRW theory: a history of the world is

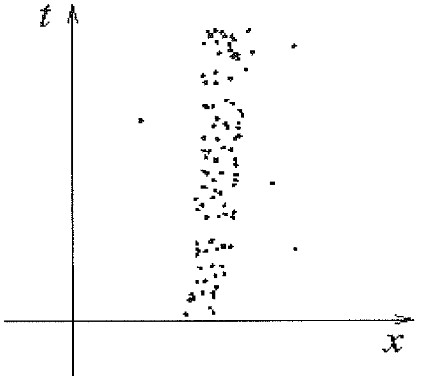

Figure 3.1: A typical pattern of flashes in space-time, and thus a possible world according to the GRWf theory.

given by a discrete set of flashes in space-time. Therefore, we should add something to the wave function also in the case of GRW theory. But this is not just an addition. Rather, these "flashes", the space-time points in which localizations occur are, in the terminology I prefer, the primitive ontology of the theory. They are what exists in space-time according to this GRW theory, that we will call GRWf or flashy GRW. This view of GRW has been adopted in (Tumulka 2006), (Maudlin 2007), (Allori et al. 2005), (Allori et al. 2007).

To sum up, Bell's idea is that GRW can account for objective reality in three-dimensional space in terms of these space-time points (X_k, T_k) that happen to correspond to the localization events (collapses) of the wave function. Note that if the number N of the degrees of freedom in the wave function is large, as in the case of a macroscopic object, the number of flashes is also large. That is to say that large numbers of flashes can form macroscopic shapes, such as tables and chairs. In Figure 3.1 a possible image of the world in this theory is shown. Note however that, at almost every time, space is in fact empty, containing no flashes and thus no matter. Thus, while classical atomic theory of matter or BMp entail that space is not everywhere continuously filled with matter but rather is largely void, this theory entails that at most times space is entirely void. As a consequence, tables and chairs are really very different from what we think they are, not being there most of the time. This, of course, applies also to ourselves.

What is the wave function in GRWf? In GRWf the space-time locations of the flashes can be read off from the history of the wave function given by (2.12) and (2.13): every flash corresponds to one of the spontaneous collapses of the wave function, and its space-time location is just the space-time location of that collapse. Accordingly, equation (2.14) gives the joint distribution of the first n flashes, after some initial time t_0. According to this theory, the world is made of flashes and the wave function serves as the tool to generate the "law of evolution" for the flashes: equation (2.11) gives the rate of the flash process, that is, the probability per unit time of the flash of label i occurring at the point x. For this reason, Roderich Tumulka, who after Bell first took seriously GRWf in his *A Relativistic Version of the Ghirardi-Rimini-Weber Model*, has preferred the word "flash" to "hitting" or "collapse center", or "jump": the latter words suggest that the role of these events is to affect the wave function, or that they are not more than certain facts about the wave function, whereas "flash" suggests rather something like an elementary, primitive event.

A final remark: It has been argued (see, e.g., (P. Lewis 2004)) that the first proposal of GRW∅ is due to Bell, on the basis of the following passage:

> No one can understand this theory until he is willing to think of [the wavefunction] as a real objective field rather than just a 'probability amplitude'. Even though it propagates not in 3-space but in $3N$-space (Bell 1987).

But the situation seems to be more complicated than that. In fact Bell subsequently proposed GRWf, as we have seen above. One could interpret that as a change of heart but, interestingly enough, after having underlined the importance of (in his terminology) local beables for a fundamental physical theory, Bell proposed GRW to be about "stuff" in configuration ($3N$-dimensional) space. Bell wrote:

> The GRW-type theories have nothing in their kinematics but the wave function. It gives the density (in a multidimensional configuration space!) of *stuff*. To account for the narrowness of that stuff in macroscopic dimensions, the linear Schrödinger

> equation has to be modified, in the GRW picture by a mathematically prescribed spontaneous collapse mechanism [Emphasis in the original] (Bell 1987).

He made a similar remark personally to Ghirardi (quoted by the latter in (Bassi and Ghirardi 2003)) in a letter dated October 3, 1989:

> As regards ψ and the density of stuff, I think it is important that this density is in the $3N$-dimensional configuration space. So I have not thought of relating it to ordinary matter or charge density in 3-space. Even for one particle I think one would have problems with the latter. So I am inclined to the view you mention 'as it is sufficient for an objective interpretation' [...] And it has to be stressed that the 'stuff' is in $3N$-space - or whatever corresponds in field theory.

It is very difficult to figure out what Bell had in mind in this respect: maybe he thought to both GRWf and GRW∅ as tenable possibilities. But this interpretation seems to be in contrast with his view of symmetry properties of the theory that led him to propose GRWf to start with, as we will see in Section 7.9.

3.2.3 Mass Density GRW

Gian Carlo Ghirardi endorsed Bell's view in spirit but, probably motivated by the development of the continuous collapse models and by the weirdness of the choice of Bell, he preferred to introduce a different proposal for his theory. Ghirardi's choice is GRW to be about a field: We have a variable $m(x,t)$ for every point $x \in \mathbb{R}^3$ in space and every time t, defined by

$$m(x,t) = \sum_{k=1}^{N} m_k \int_{\mathbb{R}^{3N}} dq_1 \cdots dq_N \, \delta(q_k - x) \left|\psi(q_1, \ldots, q_N, t)\right|^2 . \tag{3.3}$$

In words, one starts with the $|\psi|^2$-distribution in configuration space $\mathbb{R}^{3N}$, then obtains the marginal distribution of the i-th degree of freedom $q_k \in \mathbb{R}^3$ by integrating out all other variables q_j, $j \neq k$, multiplies by the mass associated with q_k, and sums over k. We will call this theory GRWm. This theory was essentially proposed by Fabio Benatti, Gian Carlo Ghirardi, and Renata Grassi in their *Describing the Macroscopic World: Closing the Circle within the Dynamical Reduction Program* (Benatti et al. 1995)[4].

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[4] They first proposed (for a model slightly more complicated than the one considered here) that the matter density be given by an expression similar to (3.3) but this difference is not relevant for our

The field $m(x,t)$ is supposed to be understood as the density of matter in space at time t. As shown in (Benatti et al. 1995), this field can be identified, on a macroscopic scale, with the usual mass density of physical objects. Since GRWm is a theory about the behavior of a field $m(x,t)$ on three-dimensional space, the microscopic description of reality provided by the mass density field, it is continuous rather than corpuscular. For example according to this description, the cat is not made of particles but she corresponds to a given configuration of the field $m(x,t)$ that, on the macroscopic scale, resembles her familiar shape. In particular, the wave function is in a superposition of the states of life and death, there is a practically instantaneous evolution of the field toward the particular distribution of masses, corresponding either to the state of life or to the state of death. This is reminiscent of Schrödinger's early view of the wave function as representing a continuous matter field. But while Schrödinger was obliged to abandon his early view because of the tendency of the wave function to spread, the spontaneous wave function collapses built into the GRW theory tend to localize the wave function, thus counteracting this tendency and overcoming the problem. Actually, given that all there is in the world is the mass density field and all the rest is determined by it, the GRW theory might even be interpreted as the only example that realized Einstein's program of developing a pure field theory that takes into account also of the quantum phenomena.

Note that the wave function in GRWm, similarly to what happens in GRWf, allows for the law of motion for the mass density field.

3.3 Many Worlds

This is the proposal according to which the wave function provides the complete description *and* it evolves according to Schrödinger's equation. If a theory like that is true, then what is the theory about?

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purposes.

3.3.1 Bare Many Worlds

There are a growing number of people that try to make sense of the many worlds theory as a theory about the wave function. The strategy is to show that the macroscopic description we provide through our everyday language somehow emerges from a description in terms of wave functions. We will call this approach MW∅.

As in cBM and GRW∅, since MW∅ is about the wave function, the first problem of this theory is to recover the perception of three-dimensional space, since physical space is configuration space, the space in which the wave function lives. This is related to the so-called *the problem of the preferred basis*: since the wave function can be written in many basis, why the one in terms of $x \in \mathbb{R}^3$ is privileged with respect to, say, the one in terms of the impulse p or the one in terms of $y \in \mathbb{R}^{1243593324}$? Proponents of this theory rely on the effect of the interaction of the system with its environment. But how this is supposed to be achieved is still a work in progress (see, again, (Wallace 2003) in this respect).

Moreover, this theory needs to explain why we assign the properties we assign to macroscopic objects if they are made of wave functions. In MW∅ properties of macroscopic objects can be determined by the eigenstate-eigenvalue (EE) rule, or what was called the fuzzy link. At the microscopic level, we have superpositions, so that microscopic objects have properties that seem contradictory to us. This is not considered problematical since, it is said, we do not have direct access to the microscopic world and therefore we cannot guarantee that the properties that we think there are are indeed there. At the macroscopic level, instead, it is argued that the different terms of the superpositions, due to the interaction with the environment, lose coherence. They "decohere", such that they are not able to interact with each other anymore so that each of them can be considered *as if* in an independent world. In other words, the idea is that each term of the superposition describes a different macroscopic object. After a given time, called decoherence time, it is practically impossible for them to interact with one another. For this reason, they are in the same space-time but they are "transparent" the one to the other.

At first, one might complain about the fact that decoherence is an approximate process (Kent 1990): for all practical purposes, the interference terms can be neglected. But the problem here is just to account for the properties of macroscopic objects in terms of the language of the wave function. This is what decoherence should be doing for us, and this is exactly what it does, since the notion of a macroscopic object is vague. For all practical purposes, we can say that a table is in the middle of the room if one of the terms of the superposition wave function is peaked in a region $R \in \mathbb{R}^3$ that corresponds to the middle of the room and the other terms will not interact with it after a while.

Chapter 4

Quantum Theories without Observer as Theories about a Primitive Ontology in Three-Dimensional Space

In the previous chapters I presented not only a variety of quantum theories without observer but also a variety of possible different metaphysics for each of them. Bohmian mechanics, GRW and the many worlds theory have been usually presented as alternative solutions of the measurement problem. But they can be interpreted very differently, depending on what the theory is considered to be about. The main suggestion here is that the different metaphysical approaches we described in the last chapter can be roughly grouped into two main categories that reflect a particular approach to physics: on one hand, we have the view according to which Bohmian mechanics, GRW and Everett theory are about some primitive ontology in three-dimensional space, while on the other hand we have the view according to which those theories should be interpreted as being about the wave function. In this chapter and in the next I will explore and compare the two approaches, and I will argue that the approach that considers the wave function as the primitive ontology of quantum theories without observer is at best problematical and that the primitive ontology of the theory, in order to be adequate, should be in three-dimensional space.

The particle trajectories in BMp (the view that Bohmian mechanics is a theory of particles), the mass density in GRWm (the viewthat GRW is about the mass density field), the flashes in GRWf (the view that GRW is about events in space-time), the particle positions in BMW (the view according to which the many worlds theory is about particles) have something in common: they are what Bell (Bell 1987) the local beables of these theories and what we have called the primitive ontology of the theory:

the stuff that things are made of [1]. *These theories are about the behavior of a primitive ontology in* $\mathbb{R}^3$, *not about the behavior of the wave function.*

The wave function also belongs to the ontology of BMp, GRWm, GRWf, BMW but it is not part of the primitive ontology: according to these theories, physical objects are not made of wave functions. The wave function is part of the ontology insofar as it can be regarded as a real entity, but it is not physical in nature. Some scholars, like Fay Dowker (Dowker and Herbauts 2006), Edward Nelson (Nelson 1985) and Bradley Monton (Monton 2002), (Monton 2006) have suggested that the wave function does not exist at all. Others, like Dürr, Goldstein and Zanghí, regard the wave function as nomological in nature. The two positions are not irreconcilable, if one is a Nominalist with respect to laws. We will come back to the issue of the nature of the wave function in Section 4.6.

Be that as it may, one of the the main *motivations* for this approach is that it seems that in each of these theories the only reason the wave function is of any interest at all is that it is relevant to the behavior of the primitive ontology. Roughly speaking, the primitive ontology tells us what matter is and the wave function tells us how the matter is moving. In BMp the wave function determines the motion of the particles *via* the guide equation, in GRWm the wave function determines the distribution of matter in the most immediate way, in GRWf the wave function determines the probability distribution of the future flashes, in BMW the wave function at a given time determines the probability distribution of the configurations at a later time.

The histories of the primitive ontology in space-time have been called by Goldstein [2] "decorations" of space-time. Each theory involves a dual structure $(\mathscr{X}, \Psi)$: the primitive ontology $\mathscr{X}$ providing the decoration, and the wave function Ψ governing its motion. The wave function in each of these theories, which has the role of generating the dynamics for the primitive ontology, has a nomological character utterly absent in the primitive ontology.

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[1] The concepts of primitive ontology and local beables could be considered as conflating for now. We will see in Chapter 6 how they differ.

[2] Private communication.

Let us note that even the Copenhagen interpretation involves a similar dual structure: what might be regarded as its primitive ontology is the classical description of macroscopic objects which Bohr insisted was indispensable (including in particular pointer orientations conveying the outcomes of experiments) with the wave function serving to determine the probability relations between the successive states of these objects. In this way, the wave function governs a primitive ontology, even for Bohr's view. An important difference, however, between the Copenhagen interpretation on the one hand and BMp, GRWm, GRWf, and BMW on the other is that the latter are fully precise about what the primitive ontology is (respectively, particle positions, continuous matter density, flashes, and positions again), whereas the Copenhagen interpretation is rather vague, even noncommittal, since the primitive ontology is macroscopic and the notion of "macroscopic" is an intrinsically vague one. Therefore, if on the one hand we acknowledge that the Copenhagen interpretation has a clear primitive ontology, on the other hand we tend to regard it as an unsatisfactory one, since it is defined on the macroscopic scale and the notion of macroscopic is ill-defined. So the natural move in order to construct a quantum theory with a clear and not vague primitive ontology would be to define the primitive ontology directly on the microscopic scale. And this is exactly what BMp, GRWf, GRWm, and BMW are doing.

4.1 Physical Equivalence

To better appreciate the concept of primitive ontology, it might be useful to regard the positions of particles, the mass density and the flashes, respectively, as the *output* of the theories presented with the wave function, in contrast, serving as part of an *algorithm that generates this output*, as emphasized in (Allori et al. 2007). Suppose we want to write a computer program for simulating a system (or a universe) according to a certain theory. For writing the program, we have to face the question: Which among the many variables to compute should be the output of the program? All other variables are internal variables of the program: they may be necessary for doing the computation, but they are not what the user is interested in. In the way we propose to understand these theories, the output of the program, the result of the simulation, should be the particle world-lines, the mass density field, respectively the flashes; the output should look like Figure 3.1. The wave function, in contrast, is one of the internal variables and its role is to implement the evolution for the output, the primitive ontology of the theory.

The proposal is that two theories be regarded as *physically equivalent* when they lead to the same history of the primitive ontology. Conversely, one could define the notion of primitive ontology in terms of physical equivalence: The primitive ontology is described by those variables which remain invariant under all physical equivalences.

4.5 Common Structure

To summarize, up to now, here are some features that the theories analyzed in this chapter have in common:

- There is a clear primitive ontology $\mathscr{X}$ that it describes matter in space and time;
- There is a wave function Ψ that evolves according to a given (possibly stochastic) dynamical evolution (either according to Schrödinger's equation or, at least, for

microscopic systems very probably for a long time approximately so).

- The wave function governs the behavior of the primitive ontology by means of (possibly stochastic) laws.

In addition, as we will see more in detail in Chapter 8, we have:

- The theory provides a notion of a typical history of the primitive ontology (of the universe), for example by a probability distribution on the space of all possible histories; from this notion of typicality the probabilistic predictions emerge.
- The predicted probability distribution of the macroscopic configuration at time t determined by the primitive ontology (usually) agrees (at least approximately) with that of the quantum formalism.

4.6 The Role of the Wave Function

Let us now clarify one issue: If the primitive ontology of the theory are the building block of the physical world, they are the stuff in three-dimensional space physical objects are made of, what is the wave function if not a material object?

4.6.1 The Wave Function as a Property?

One way of interpreting the wave function if it is not part of the primitive ontology is to say that the wave function is a property of the particles. Monton seems to have this view in some of his writings:

> the wave function doesn't exist on its own, but it corresponds to a property possessed by the system of all the particles in the universe (Monton 2006).

If it is the case, then the wave function is not physical but it is instead an abstract entity. It is not really clear to me what "the wave function is a property" is supposed to mean, given that it is not clear to me what a property is supposed to be.

Be that as it may, what kind of property is the wave function supposed to be? Categorical or dispositional? In my understanding, a dispositional property is a property that is what it is in virtue of the laws of physics. For example, the mass of an object can be considered a dispositional property in the sense that it expresses the resistance of the body to be accelerated by external forces. In contrast, the mass can be thought as a categorical property of the body as it specifies its own nature. It seems difficult to regard the wave function as property *of the particle* in any case, since the wave function is *of the universe*, the biggest system of all. We have seen that in Bohmian mechanics, since we have particles, we can define a wave function for the subsystem of configuration x using the conditional wave function $\psi(x) = \Psi(x, Y)$, where Y is the actual configuration of the environment of the x-system. In any case, it does not seem right to consider the wave function (not even the conditional one) as a categorical property of the particles: in fact, it does not in any ways determine its nature. It might seem a little less far fetched to think to the conditional wave function as a dispositional property but actually it is difficult since it might happen not: the conditional wave function might not evolve according to Schrödinger's equation. It would do that only in particular situations like the one in which the wave function has a particular form, the so called effective wave function. In any case, independently of whether one can make sense of the wave function being a property of the particle or one has to assume that the wave function is an holistic property, a property of the universe as a whole, I do not really see any advantage in saying that the wave function is a property, unless what one means is, at the end of the day, that it is a law. Laws and properties seems to share the feature of being abstract entities, so one would not worry about the wave function being physically real either way. But saying that the wave function is a law gives a clearer role to the wave function than saying it is a property. Talking of physical theories in terms of properties rather than in terms of laws does not seem to do any good. One might think that there are categorical properties that determine the very nature of the physical objects. My intuition is just the other way round: a particle is considered to be an electron rather than a proton because, say, in the bubble chamber it will rotate one way or another under the magnetic field imposed on it. That is, physical objects and their properties are parasitic on the theory in which they are described, rather the other way round. Therefore, it seems to proceed backwards to say that the wave function is a property rather than saying that it is a law.

4.6.2 The Wave Function as a Useful Mathematical Tool

Another proposal, or maybe a different way of intending the former proposal, is to consider the wave function just as a useful mathematical tool. Monton writes:

> I think that the wave function is a useful mathematical tool; it is useful to describe systems as having quantum states, represented by wave functions. But as a matter of ontology, the wave function doesn't exist or at least, the wave function is no more real than the numbers (like 2, or B) that go into the equations used to describe quantum systems. The wave function is, at best, an abstract entity and if you're a nominalist about abstract entities like I am, then you should be happy to say that the wave function doesn't exist (Monton 2006).

A similar view has been discussed by Fay Dowker and Isabelle Herbauts (Dowker and Herbauts 2006) in the framework of GRW theory: They provide a model of spontaneous collapse theories on a lattice in which

> the state $|\psi_0\rangle$ can be deduced FAPP [for all practical purposes] from the field configuration [...] and becomes an"executive summary" of the past reality containing no independent information.

A number of objections have been raised to the view that the wave function is not existing as a physical object, which are similar to the objections that have been raised against the next proposal that we will analyze. For this reason, we will deal with them in the next sections.

4.6.3 The Wave Function as a Law

A slightly different but related approach has been proposed by Dürr, Goldstein and Zanghì who have analyzed the nature of the wave function in the framework of Bohmian mechanics in their *Bohmian Mechanics and the Meaning of the Wave Function* (DGZ 1997), and that I have taken as more or less implicitly up to now.

In their view the wave function has to be intended as the object that allows for the generation of the law of motion for the particles rather than a field on configuration space or just a mathematical tool. As they say:

> the role of the wave function in this theory, expressed by the association $\psi \rightarrow v^{\psi}$, is to generate the vector field, given by the right hand side of (3) [the guide equation], that defines the motion (DGZ 1997).

In this theory, the wave function has to be intended not as a part of physical reality but as a law:

> We propose that the wave function belongs to an altogether different category of existence than that of substantive physical entities, and that its existence is nomological rather than material (DGZ 1997).

The wave function expresses a law for the motion of the particles, just as the Hamiltonian in classical mechanics is the generator of the motion of the positions of the particles. In fact, in classical mechanics, the complete description of any physical system is provided by $X = (q_1, ..., q_N, p_1, ..., p_N)$, the set of the configurations and the momenta of all the particles. Then the classical Hamiltonian H_{class} is a function of the space of the Xs, the phase space, and it is the generator of the time evolution of that state:

$$\frac{dq}{dt} = \frac{\partial H_{class}}{\partial p} \tag{4.14}$$

$$\frac{dp}{dt} = -\frac{\partial H_{class}}{\partial q}, \tag{4.15}$$

or, more compactly,

$$\frac{dX}{dt} = \mathrm{Der} H_{class}, \tag{4.16}$$

where Der is a suitable operation of derivation. In the same way logΨ is the generator for the motion of particles:

$$\frac{dQ}{dt} = \mathrm{Der}[\log\Psi]. \tag{4.17}$$

In this framework, Bohmian mechanics is a theory about particles that evolve according to a law of motion depending on the wave function, that has to be intended, according to Dürr, Goldstein and Zanghí, as part of the law for the particles. As in Newtonian mechanics it is necessary to specify the momentum p and the potential V in order to generate the trajectories $q(t)$ of the particles, so in BMp it is necessary to specify the wave function ψ and the Hamiltonian H in order to generate the trajectories $q(t)$ of the particles. The situation is very similar in the plethora of theories discussed in 4.3:

In Bohmian mechanics the primitive ontology (particles positions) evolves deterministically according to a law generated by a wave function that evolves deterministically; in BMW the primitive ontology (particles positions) evolves stochastically according to a law generated by a wave function that evolves deterministically; in BTQFT, and in in stochastic mechanics the primitive ontology (particles positions) evolves stochastically according to a law generated by a wave function that evolves deterministically; in GRWp the primitive ontology (particles positions) evolves stochastically according to a law generated by a wave function that evolves stochastically; in Sm the primitive ontology (the mass density field) evolves deterministically according to a law generated by a wave function that evolves deterministically; in GRWm the primitive ontology (the mass density field) evolves stochastically according to a law generated by a wave function that evolves stochastically; in Sf the primitive ontology (the flashes) evolves stochastically according to a law generated by a wave function that evolves deterministically; in GRWf the primitive ontology (the flashes) evolves stochastically according to a law generated by a wave function that evolves stochastically; in BQFT the primitive ontology (fields) evolves deterministically according to a law generated by a wave function that evolves deterministically. In all these cases, the primitive ontology $\mathscr{X}$ evolves according to a given equation governed by the wave function. Therefore, again, it is necessary to specify the wave function ψ in order to generate the trajectories of the primitive ontology. Therefore, according to Dürr, Goldstein and Zanghí we can conclude that the wave function in all those theories has a sort of nomological status that differs from the one of the primitive ontology: the wave function has the role of generating the dynamics for the primitive ontology, whatever it is.

One complaint might be that this view suggests that there are different degrees of reality, as suggested by Harvey Brown and David Wallace in their *Solving the Measurement Problem: de Broglie-Bohm Loses out to Everett* (Brown and Wallace 2005) and by David Albert [4]: since the primitive ontology composes physical objects while the wave function does not, either we deny the existence of the wave function or we have to admit that something is more real than something else.

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[4] Private communication.

This seems to me an unfair criticism. Saying that the wave function is real but not physical does not imply there are different degrees of reality: in fact, they might be two kind of substances, or entities, or different way of existing. After all, the very same objections could be raised (but they are not) to a Platonist in the philosophy of mathematics, a dualist in the philosophy of mind, and a realist with respect to laws in ethics or in philosophy of science: Numbers, consciousness, moral and physical laws can exist even if they are non-physical.

Another objection to this view is the following, as expressed by Harvey Brown and David Wallace in the framework of Bohmian mechanics:

> [...] reality is not some property which we can grant or withhold in an arbitrary way from the components of our mathematical formalism. The wave-function evolves; it dynamically influences the corpuscles; in interference experiments its existence is explanatorily central to the observed phenomena. On what grounds could we just dismiss it as a mathematical fiction? (Brown and Wallace 2005)

There are several components to this idea: first, the wave function has an explanatory role; second, the wave function interacts with the particles; third, the wave function evolves in time. For these reasons, they argue, we should consider the wave function as physically real.

As already stressed, I do not think that the claim that the wave function should be physically real because it is explanatory is really a serious objection: even if something has to be postulated existing in order to explain the behavior of matter it does not follow that that entity has to be *physically* real. In fact, if one has a realistic view of laws, laws are explanatory but they do not exist in physical space. Of course the problem would be to explain the somehow mysterious interaction between laws and matter, but this is a general problem for the view, not a particular one for quantum theories without observer.

Concerning the second objection, that since the wave function interacts with the particles in Bohmian mechanics it should be considered physical, one could respond saying that the situation is similar to the one in classical mechanics: as in Newton's theory the potential can interact with the particles and no one would be tempted to say that the potential is physically existing, so in quantum mechanics the wave function interacts with the particles but it is not necessary to assume it to be a physical field. Wallace and Brown anticipate this reply:

> [...] there is a bad analogy to resist here. From the corpuscles perspective, the wave-function is just a (time-dependent) function on their configuration space, telling them how to behave; it superficially appears similar to the Newtonian or Coulomb potential field, which is again a function on configuration space. No-one was tempted to reify the Newtonian potential; why, then, reify the wave-function? (Brown and Wallace 2005)

and respond emphasizing that the analogy between the wave function and the potential is a bad one:

> Because the wave-function is a very different sort of entity. It is contingent (equivalently, it has dynamical degrees of freedom independent of the corpuscles); it evolves over time; it is structurally overwhelmingly more complex (the Newtonian potential can be written in closed form in a line; there is not the slightest possibility of writing a closed form for the wave-function of the Universe.) (Brown and Wallace 2005)

That is, the wave function evolves in time in a way which is independent of the particles, while the potential in classical mechanics does not. In addition, the wave function cannot be written, contrarily to what can be done with the potential, in a closed form in a line. Therefore, the wave function and the potential are not even similar mathematically. The problem is therefore that the analogy between the wave function and the Hamiltonian is not that straightforward. Brown and Wallace continue making a contrast between the potential in classical mechanics on one hand and the electromagnetic fields in classical electrodynamics and the wave function in quantum theories without observer on the other hand. The wave function should be regarded to be similar to the electromagnetic fields since they are both evolving in time according to an independent equation, and should be contrasted with the potential that does not. While the potential was not considered as physically real, the electromagnetic fields have been. As a consequence, also the wave function should be considered as physically real. In their words:

> Historically, it was exactly when the gravitational and electric fields began to be attributed independent dynamics and degrees of freedom that they were reified: the Coulomb or Newtonian fields may be convenient mathematical fictions, but

> the Maxwell field and the dynamical spacetime metric are almost universally accepted as part of the ontology of modern physics (Brown and Wallace 2005).

I will come back to the status of the electromagnetic fields in Chapter 6. Be that as it may, let me notice that John Wheeler and Richard Feynman in their *Interaction with the Absorber as the Mechanism of Radiation* (Wheeler and Feynman 1945) tried to eliminate the electromagnetic field in their theory, rewriting the dynamical equations such that the electromagnetic field entirely disappears. One could try the same in the case of the wave function in order to be able to say that the wave function is not physical. Goldstein and Teufel (Goldstein and Teufel 2000) try to find a way to eliminate it from the theory and recover it as an effective, phenomenological object just as Wheeler and Feynman attempted to do for the electromagnetic field. The idea starts from some observations in quantum cosmology. In that theory, the basic equation for the wave function is the Wheeler-De Witt equation. This is an equation for the wave function of the universe Ψ that in this case is static and can be written as follows:

$$H\Psi = 0 \tag{4.18}$$

for a suitable Hamiltonian H. Similarly to what happens in Bohmian mechanics, also in quantum cosmology it would be possible to regard Ψ as the generator of the velocity field v^{Ψ} for the primitive ontology $\mathscr{X}$ of quantum cosmology:

$$\frac{d\mathscr{X}}{dt} = v^{\Psi} \tag{4.19}$$

where in this case $\mathscr{X}$ would include also the configurations of the gravitational field. The field form would be different from the one of Bohmian mechanics and, again differently from Bohmian mechanics the wave function Ψ would not evolve in time. The basic idea is that one could recover the Schrödinger's equation describing a time dependent wave function phenomenologically, being the Wheeler-De Witt equation the fundamental law.

This is connected with the third criticism that the wave function evolves in time, while we tend to think of laws as constant in time. While at the present state of development Bohmian mechanics it might be uncomfortable to regard the wave function as a law because of this time dependence, as suggested by Brown and Wallace, in a future quantum cosmology physical reality would be described by the relevant primitive ontology $\mathscr{X}$ and the wave function would be static and easily considerable a law.

To summarize, the present situation is that the wave function evolves in time but in a more general theory the hope is that this would not be the case anymore. As Shelly Goldstein often stresses [5], here is a nice graphical summary of how the situation has evolved historically in quantum mechanics:

$$(\psi) \rightarrow (\mathscr{X}, \psi) \rightarrow (\mathscr{X}).$$

We started from standard quantum mechanics, in we just had the wave function; we moved to Bohmian mechanics, in which matter is made of particles and the wave function is a time dependent object that tells matter how to move; and we will (probably, or hopefully) end up with a theory of quantum cosmology with a suitable primitive ontology $\mathscr{X}$ in which it would be straightforwardly possible to regard the wave function as a law, it being static.

In any case, it is argued, even if we might have a static wave function, the two of the three features that distinguish the wave function from the potentials are still there:

> As for contingency, it is at most an article of faith with some physicists that the Wheeler-de Witt equation has a unique solution. And as for complexity (in our view perhaps the most important criterion) the structure encoded in the cosmological wavefunction will if anything be richer than that encoded in the nonrelativistic wavefunction (Brown and Wallace 2005).

As far as the lack of simplicity of the wave function, I wish to observe that, as soon as one writes the equation for the wave function as (4.18), some sort of relevant simplicity is already recovered, that equation expressing the fact that there exists an Hamiltonian H such that the wave function Ψ is static. In the case of the worry about the uniqueness of the solution of the Wheeler-de Witt equation, I do not see any reason to be that pessimistic as to believe that there will not be a unique solution: there are no reason to believe the solution will be unique, but there might be no reason also to

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[5] Private communication.

indicate that it will not be it as well.

Another objection, raised again by Brown and Wallace, to the idea that the wave function could be seen as a law in perspective of the development of a theory of quantum cosmology is that this project is purely speculative and, in any case, in the theory that we have right now the situation is different: the wave function does evolve in time and this makes it very different from what we would consider a law. In this regard, I would like to observe that we should always take theories in perspective, keeping in mind that in a sense they are always speculative and provisional, since they are not logical deduction from experimental data. So, the charge does not seem to be that devastating:. In the present theory we cannot so naturally or straightforwardly identify the wave function as a law because it evolves in time. But why do we worry so much? What we do when we construct our fundamental physical theories is try to get a grasp on what there is, and the way in which we proceed is by little steps forward, one at the time. What the theory that we have right now is telling us is that the wave function seems to play a certain role, namely the generator of the motion of matter. Given that in the current theory it evolves in time, the wave function might not fit perfectly with our intuition that laws are static, but the fact that the wave function is not static in the present theory is telling us is not so much that the wave function is a material object but rather that we should look for a theory in which the wave function is static. In fact, what seems to be the lesson of the quantum theories without observer is that the wave function in the theory has a role which is different from the one of the primitive ontology.

A last complaint could be that the wave function is controllable [6]: we can prepare physical systems in the state that we want. If so, it is difficult to regard the wave function as a law, since we do not seem to have control over them. This objection is easily taken care of if one remembers that the wave function that we can have control over is the wave function of the system, while the one that should be intended as nomological is rather the wave function of the universe, and we do not have any control

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[6] Shelly Goldstein mentioned this objection to me, together with its response.

over it.

4.7 State, Primitive Ontology and Bell's Alternatives

Usually people talk about states, while I have drawn my attention to the primitive ontology. It is crucial to contrast the two notions. The state of the system is what provides a complete description of the world to any instant in time. It can be said to be the ontology. In the language I used, it is therefore composed of the variable characterizing the primitive ontology supplemented by all those variables that allow for the closure of its dynamics. As we have seen above, in order to determine the evolution of the primitive ontology in time, we need to specify its law of evolution, that in quantum theories without observer depends on the wave function, which also evolves in time.

Consider BMp: the primitive ontology is the configuration Q of particles in space. In contrast, the state is given by the couple (Q, Ψ) of the positions of all the particles and the wave function of the universe. Similarly, in BMW the primitive ontology is the one of configurations and the state is the couple (Q, Ψ). In the case of GRWp, the particles have trajectories determined by an evolution generated by a GRW-evolving wave function. In SM, the particles have stochastic trajectories generated by a Schrödinger evolving wave function. That is also true for BTQFT, for a different stochastic law for the trajectories. In BQFT, Schrödinger wave function generates deterministic trajectories for the fields. In the case of GRWm, the primitive ontology is given by the mass density field $m(x,t)$, whose evolution is governed by a stochastic wave function. Note that in this case the mass density, as it is defined, is a functional of the wave function: $m = F(\Psi)$, with F given by equation (3.3). The same can be said in the case of GRWf: the probability distribution of the flashes, that are the primitive ontology of the theory, are defined in terms of the wave function by equation (2.14). The situation is the same in the case of Sm and Sf, that differ from GRWm and GRWf respectively only in the fact that the wave function evolves linearly.

Therefore, this is the crucial difference between GRWm, GRWf, Sm and Sf on one hand and BMp, SM, GRWp, BMW and BTQFT on the other: while in the latter the state there is composed of the primitive ontology and by the wave function, in the former there is a sense in which this is not the case.

Therefore, on one hand the state by definition gives us everything. But on the other hand also in these theories of mass density and flashes the state tells us less than the primitive ontology because it does not specify *per se* which of the variables has a nomological role (the wave function ψ) and which constitute physical objects (the primitive ontology).

In this framework, the measurement problem is the problem of the inadequacy of the wave function as the primitive ontology of quantum mechanics. The measurement problem is caused by the fact that the wave function cannot represent physical objects. So possible solutions of the measurement differ not in the fact that we either add something or we let the wave function evolve to an equation that is different form Schrödinger. Rather, different solutions are characterized by what they take as the histories of the primitive ontology of the theory. Therefore, we could label a theory a "Bohmian" solution of the measurement problem if the wave function and the primitive ontology are independent. According to this definition, SM, GRWp, BMW, and BTQFT are, together with BMp, are "Bohmian solutions" of the measurement problem. Instead, we can say that a "GRW" strategy to solve the measurement problem is the one in which the primitive ontology depends on the wave function: this is the case of GRWm, GRWf, Sm and Sf, that all have in common the fact that their primitive ontology is defined in terms of the wave function. Both the flashes and the mass density in these theories are functionals of the wave function. Therefore in BMp, SM, BMW and BTQFT the state is given by $(\mathscr{X}, \Psi)$. This is also the case in GRWf, GRWm, Sf and Sm, where now $\mathscr{X} = F(\Psi)$. In fact, in the latter theories the wave function, strictly speaking, is not the state of the system anymore, if we have to intend "state" as what we need to specify in order to provide a complete description. In fact, there is nothing in Ψ that allows to determine the function F that specifies the primitive ontology.

4.8 Supervenience: Logical and Natural

In philosophical jargon, when there is dependence between two variables it is said that the dependent one supervenes of the other. The template for the definition of supervenience is the following: Y supervene on X if no two possible situations are indiscernible with respect to X while differing in Y. For instance, chemical properties supervene on physical properties insofar as any two possible situations that are physically indistinguishable are chemically indistinguishable. One could notice that the mass density and the flashes supervene on the wave function: there cannot be a difference in the mass density or in the flashes without a difference in the wave function. As we saw, this is very different to what happens in Bohmian mechanics, in which positions are specified independently from the wave function. The mass density and the flashes, unlike the positions in BMp, are not be specified in addition to the wave function, but rather are determined by it. Therefore, they are in some respect "hidden variables" of the theories.

Peter Lewis (P. Lewis 2006) has argued that this is an indication of the fact that there is no mass density or flashes ontologically "added" to the wave function, they are just rules of translation from the language of the wave function to ordinary language. In the case of the flashes, he writes:

> the structure of flashes is already present in the wavefunction; flashes are discontinuous localizations in the wavefunction, projected into three-dimensional space. So again, there is no need to postulate the flashes as an addition to our fundamental ontology (P. Lewis 2006).

But this unspecified supervenience it is not enough to arrive to Lewis' conclusion, as we will now see. More precise notions of supervenience can be obtained by filling in this template.

A specification of supervenience that is relevant for our purpose is connected with the distinction between logical (or conceptual) and natural (or nomic) supervenience. Y supervenes logically on X if no two logically possible situations are identical with respect to X but distinct with respect to Y. In its *The Conscious Mind* David Chalmers writes:

> One can think of it loosely as possibility in the broadest sense, corresponding roughly to conceivability, quite unconstrained by the laws of our world. It is useful to think of a logically possible world as a world that it would have been in God's power (hypothetically!) to create, had he so chosen. [...] In determining whether it is logically possible that some statement is true, the constraints are largely conceptual. [...] It should be stressed that the logical supervenience is not defined in terms of deducibility in any system of formal logic. Rather, logical supervenience is defined in terms of logically possible worlds (and individuals), where the notion of a logically possible world is independent of these formal considerations. [...] biological properties supervene logically on physical properties. Even God could not have created a world that was physically identical to ours but biologically distinct. There is simply no logical space for the biological facts to independently vary. When we fix all the physical facts about the world including the facts about the distribution of every last particle across space and time we will in effect also fix the macroscopic shape of all the objects in the world, the way they move and function, the way they physically interact (Chalmers 1996).

In general, when we have logical supervenience between Y and X, we say that X entails or implicates Y, i.e. $Y = f(X)$. To provide an example, the description "table" supervenes logically on the configuration of the particles composing the table: the table *is just* a bunch of particles.

We can also have a supervenience which is not logical: this is the case in which X is always correlated with Y in the natural world. Here is the example that Chalmers provide:

> the pressure exerted by one mole of a gas systematically depends on its temperature and volume according to the law $pV = KT$, where K is a constant. [...] In the actual world, whenever there is a mole of gas at a given temperature and volume, its pressure will be determined: it is empirically impossible that two distinct moles of gas could have the same temperature and volume, but different pressure. It follows that the pressure of a mole of gas supervenes on its temperature and volume in a certain sense. [...] But this supervenience is weaker than logical supervenience. It is logically possible that a mole of gas with a given temperature and volume might have a different pressure; imagine a world in which the gas constant K is larger or smaller, for example. Rather, it is just a fact about nature that there is this correlation. This is an example of natural supervenience of one property on others: in this instance, pressure properties supervene naturally on temperature, volume, and the property of being a mole of gas (Chalmers 1996).

Another example of natural supervenience is this case with respect to entities in a fundamental physical theory, is the one between the charge density ρ and the electromagnetic fields, as pointed out by Tim Maudlin in his *Completeness, Supervenience, and Ontology* (Maudlin 2006). The relation between the two is

$$\rho = \frac{1}{\epsilon_0} \nabla \cdot E \tag{4.20}$$

and it is a law of nature. In general, Y supervenes naturally on X if any two naturally possible situations with indiscernible X have indiscernible Y. A naturally possible situation is one that could actually occur in nature, without violating any natural laws: this is a much stronger constraint than logical possibility. We can think of this kind of possibility as nomic possibility: possibility subject to the laws of nature.

The distinction between logical and natural supervenience can be summarized as follows: if Y supervenes logically on X, then once God has created a world with certain X, the Y comes along for free; if Ysupervenes naturally on X, then after making the X, God had to do more work in order to make the Y: he had to make a law relating the X and the Y. Once the law is defined, X will automatically bring along the Y. But one could, in principle, have had a situation where they did not.

As we have seen above, for example in BMp the state does not supervene on the primitive ontology. In the case of GRWm, GRWf, Sm and Sf insetad the primitive ontology is determined by, supervenes on, the wave function. But what kind of supervenience is that? The primitive ontology naturally, and not logically, supervene on the wave function. That is, it is an additional law of nature. There is nothing that forces us conceptually to choose one law or another, one needs to *posit* one. We will discuss this issue more in Chapter 5.

4.10 What is the Wave Function?

Before concluding this chapter, I wish to add a further remark. As we have seen previously, in quantum mechanics the wave function is a ray. That is, ψ and $c\psi$, where c is a nonzero complex number, represent the same physical state. Why we should believe the wave function to be such a mathematical object? As emphasized by Wigner (Wigner 1939), in ordinary quantum mechanics both ψ and by $c\psi$ generate the same transition probability, and therefore, they do not represent physically distinct states. If one considers these probabilities as fundamental, like the Copenhagen interpretation does, then it makes sense that both ψ and $c\psi$ describe the same physical state. In Bohmian mechanics the two wave functions generate the same velocity field for the particles, and in GRW, the same can be noticed (see (Allori et al. 2007)). In all cases, ψ and $c\psi$ describe the same physics, and this justifies the mathematical nature of the wave function as a ray.

This suggests also that the wave function is like a gauge potential. It is similar in this respect to the A and ϕ gauge fields that appear in classical electrodynamics. That is, the wave function is what generates the correct histories of the primitive ontology, just like A and ψ generates in classical electrodynamics. Indeed, since the wave function a ray, it is defined up to a time gauge: ψ and $c\psi$ represents the same state and c can depend on time. This expresses a sort of temporal gauge invariance. From quantum mechanics we know that the time evolution of the wave function is given by a linear equation in Hilbert space, namely Schrödinger's equation. But this is only a matter of convenience and not justified by the geometrical nature of the entity that represents the state. In other words, Schrödinger's equation gives the evolution of the wave function only in a particular time gauge, the one in which the time evolution is linear. To put it differently, the wave function does not necessarily have a deterministic behavior, being it similar to a classical gauge field, until one fixes the time gauge. As in classical electrodynamics one chooses a gauge in order to fix the potentials, the same has to be done in quantum mechanics such that it is possible to write a deterministic (linear) equation for the wave function. One difference between quantum mechanics and classical electrodynamics is that while in classical electrodynamics Maxwell's equations of motion (in addition to Newton's law) are for the electromagnetic fields E and B and not in terms of their potentials A and ϕ, in quantum mechanics instead just the opposite is true. The fundamental equation (the only one there is) is in terms of the gauge field ψ. The gauge invariant objects are defined in term of the wave function: in ordinary quantum mechanics this gauge invariant quantity is the transition probability, in Bohmian mechanics instead the invariance quantity is velocity field. In the other theories it would be the quantity that defines the histories of the primitive ontology.