

IZVOD Q-VEKTORA (\vec{Q}) PREKO ω -JEDNADŽBE

- kvadrirana stroficheska ω -jednadžba:

$$\left(f_0 \frac{\partial^2}{\partial n^2} + \sigma \nabla^2 \right) \omega = \frac{\partial}{\partial n} \mathcal{F}(\phi, \frac{1}{f_0} \nabla^2 \phi + f) + \nabla^2 \mathcal{F}(\frac{1}{f_0} \phi, -\frac{\partial \phi}{\partial n}) \quad (*)$$

$\nabla^2 \dots$ "fizički Laplasijan"

- promatrajmo ω na nekoj uspravnoj plohi i pridaje li smo joj pravilnu sinusnu biperiódichku funkciju (za lakšu interpretaciju):

$$\omega = \omega_0 \sin(kx) \sin(ly)$$

$$\Rightarrow \nabla^2 \omega \sim -(k^2 + l^2) \omega \sim -\omega \sim \omega$$

- dakle, cijela desna strana ω -jednadžbe je proporcionalna ω i vertikalnom komponentom brine ω (u x, y, z sustavu)

- sada možemo pisati sljedeće:

$$\omega \sim \mathcal{F}\left(\frac{\partial \phi}{\partial n}, \frac{1}{f_0} \nabla^2 \phi + f\right) + \mathcal{F}\left(\phi, \frac{1}{f_0} \frac{\partial}{\partial n} \nabla^2 \phi\right) + \mathcal{F}\left(\frac{1}{f_0} \nabla^2 \phi, -\frac{\partial \phi}{\partial n}\right) + \mathcal{F}\left(\frac{1}{f_0} \phi, -\frac{\partial}{\partial n} \nabla^2 \phi\right)$$

RASPISANA DESNA STRANA U (*)

$$\Rightarrow \omega \sim \mathcal{F}\left(\frac{\partial \phi}{\partial n}, \frac{1}{f_0} \nabla^2 \phi + f\right) + \mathcal{F}\left(\phi, \frac{1}{f_0} \frac{\partial}{\partial n} \nabla^2 \phi\right) + \mathcal{F}\left(\frac{\partial \phi}{\partial n}, \frac{1}{f_0} \nabla^2 \phi\right) - \mathcal{F}\left(\frac{1}{f_0} \phi, \frac{\partial}{\partial n} \nabla^2 \phi\right)$$

$$\Rightarrow \omega \sim \mathcal{F}\left(\frac{\partial \phi}{\partial n}, \frac{2}{f_0} \nabla^2 \phi + f\right)$$

- kako razjehati: $\frac{1}{f_0} \nabla^2 \phi = \xi \vec{g}$

$$\Rightarrow \omega \sim \mathcal{F}\left(\frac{\partial \phi}{\partial n}, 2\xi \vec{g} + f\right) = \vec{k} \cdot \left[\nabla \left(\frac{\partial \phi}{\partial n} \right) \times \nabla (2\xi \vec{g} + f) \right] =$$

dola

$$= (\vec{k} \times \nabla \frac{\partial \phi}{\partial n}) \cdot \nabla (2\xi \vec{g} + f) = \frac{\partial}{\partial n} (\vec{k} \times \nabla \phi) \cdot \nabla (2\xi \vec{g} + f) \dots$$

- znamo: $\vec{V}_g = \frac{1}{f_0} \vec{k} \times \nabla \phi$

... $w \sim f_0 \frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla (2\xi_g + f) = 2f_0 \frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla \xi_g + f_0 \frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla f$

- sada: $\frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla \xi_g = \nabla \cdot (\xi_g \frac{\partial \vec{V}_g}{\partial \eta}) - \xi_g \nabla \cdot \frac{\partial \vec{V}_g}{\partial \eta} = \textcircled{A} - \textcircled{B}$

$\textcircled{A} 2f_0 \frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla \xi_g = 2f_0 \left[\nabla \cdot (\xi_g \frac{\partial \vec{V}_g}{\partial \eta}) - \xi_g \nabla \cdot \frac{\partial \vec{V}_g}{\partial \eta} \right] =$

- to uostavimo gore

$= 2f_0 \left[\nabla \cdot (\xi_g \frac{\partial \vec{V}_g}{\partial \eta}) - \xi_g \frac{\partial}{\partial \eta} \nabla \cdot \vec{V}_g \right] =$

$= 2f_0 \nabla \cdot (\xi_g \frac{\partial \vec{V}_g}{\partial \eta})$

$\textcircled{B} f_0 \frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla f = f_0 \frac{\partial}{\partial \eta} \left(\frac{1}{f_0} \vec{k} \times \nabla \phi \right) \cdot \nabla f = \vec{k} \times \nabla \frac{\partial \phi}{\partial \eta} \cdot \nabla f =$

- kako unijedi: $\frac{\partial \phi}{\partial \eta} = -\alpha = -\frac{RT}{\mu}$

$\Rightarrow f_0 \frac{\partial \vec{V}_g}{\partial \eta} \cdot \nabla f = -\frac{R}{\mu} \vec{k} \times \nabla T \cdot \nabla f = -\frac{R}{\mu} \left[\vec{k} \times \left(\frac{\partial T}{\partial x} \vec{i} + \frac{\partial T}{\partial y} \vec{j} \right) \right] \cdot \nabla f =$

$= -\frac{R}{\mu} \left[\frac{\partial T}{\partial x} \vec{j} - \frac{\partial T}{\partial y} \vec{i} \right] \cdot \left(\frac{\partial f}{\partial x} \vec{i} + \frac{\partial f}{\partial y} \vec{j} \right) = -\frac{R}{\mu} \beta \frac{\partial T}{\partial x}$

$\Rightarrow w \sim \textcircled{A} + \textcircled{B} = 2f_0 \nabla \cdot (\xi_g \frac{\partial \vec{V}_g}{\partial \eta}) - \frac{R}{\mu} \beta \frac{\partial T}{\partial x}$

- dimenzionalna analiza pokazuje da je $\textcircled{B} \ll \textcircled{A}$ i to iz 2 razloga:

- 1) $\beta \sim 10^{-12}$
- 2) za planetarna cirkulacija se temperatura po paraleli (po x) znatnije mijenja nego po meridijanu (po y)

- da budemo sigurni, koordinatni sustav postavimo tako da je os x paralelna vjetrovanju $\Rightarrow \frac{\partial T}{\partial x} = 0 \Rightarrow \textcircled{B} = 0!$

- sada uostavimo pokretu (Q-vektor): $\vec{Q} = -f_0 \xi_g \frac{\partial \vec{V}_g}{\partial \eta}$

$\Rightarrow \boxed{w \sim -2 \nabla \cdot \vec{Q}}$

