Gravitacija kao specijalna relativistička teorija polja

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Uvod

Svojstva gravitacije

- dugodosežna interakcija → graviton je bezmasena čestica
- statička sila → graviton je bozon
- nije opaženo negativno međudjelovanje → spin je paran
- opaženo ogibanje svjetlosti kraj zvijezda → spin nije 0

- propagator čestice s m=0, s=0 ~ k^{-2}
- jedino vezanje: $T^{\mu}_{\mu} k^{-2} T^{\nu}_{\nu}$
- ali $T^{\mu}_{\mu} \doteq 0$. u elektromagnetizmu!

→ **GRAVITON** : masa =0, spin =2

SRTP spina 1

- bezmaseno vektorsko polje

A_μ → $\partial^2 A^\mu = j^\mu$

4 komponente, 2 stanja heliciteta ↔ gustoća energije nije poz. definitna

- rješenje: Lorentzov uvjet $\partial_\mu A^\mu = 0$ → očuvanje struje $\partial_\mu j^\mu = 0$
- želimo lagranžijan takav da iz jednadžbi gibanja slijedi očuvanje struje $D^\mu(A) = j^\mu$

$\partial_\mu D^\mu(A) = 0$

BAŽDARNI UVJET

- najopćenitiji LI lagranžijan: $\mathcal{L} = \alpha \partial_\mu A_\nu \partial^\mu A^\nu + \beta \partial_\mu A_\nu \partial^\nu A^\mu$
- jednadžba gibanja: $D^\nu(A) = \alpha \partial^2 A^\nu + \beta \partial_\mu \partial^\nu A^\mu = 0$

→ uvrstimo u baždarni uvjet : $\alpha = -\beta$

$$\mathcal{L} = -\frac{1}{2}\partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2}\partial_\mu A_\nu \partial^\nu A^\mu = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D^\nu = \partial_\mu F^{\mu\nu}$$

- baždarna invarijantnost: $\delta \mathcal{L} = \left[\frac{\delta \mathcal{L}}{\delta A_\nu} - \partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu A_\nu)} \right] \delta A_\nu$

$$= \partial_\mu F^{\mu\nu} \delta A_\nu \sim \partial_\mu \partial_\nu F^{\mu\nu} \Lambda$$

$$\delta A_\mu = \partial_\mu \Lambda$$

Vezanje na materiju

- slobodni lagranžijan

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- jednadžbe gibanja

$$\partial^2 \phi = 0$$
$$\partial^2 \phi^* = 0$$
$$\partial_\mu F^{\mu\nu} = 0$$

baždarne transformacije

$$\delta A_\mu = \partial_\mu \Lambda$$ lokalna transformacija

$$\delta \phi = ie\lambda \phi$$ globalna transformacija

- varijacija lagranžijana s obzirom na λ :

$$\delta_\lambda \mathcal{L}_0 = -\frac{ie}{2}\partial_\mu \lambda \left(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*\right)$$
$$= e\lambda \partial_\mu j^\mu_{(0)}$$

$$j^\mu_{(0)} = -\frac{i}{2}\left(\phi \partial^\mu \phi^* - \phi^* \partial_\mu \phi\right)$$

Noetherina struja

- vrijedi zakon očuvanja $\partial_\mu j^\mu_{(0)} = 0$

- ali $j^{\mu}_{(0)}$ nije izvor za $F^{\mu\nu}$ → lagranžijanu dodajemo član koji reproducira Maxwellovu jednadžbu $\partial_\mu F^{\mu\nu} = -ej^{\nu}_{(0)}$

⇒ $$\mathcal{L}_1 = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + eA_\mu j^{\mu}_{(0)}$$

jednadžbe gibanja:
$$\partial^2\phi + ie\partial_\mu A^\mu\phi + 2ieA_\mu\partial^\mu\phi = 0$$
$$\partial^2\phi^* - ie\partial_\mu A^\mu\phi^* - 2ieA_\mu\partial^\mu\phi^* = 0$$
$$\partial_\mu F^{\mu\nu} = -ej^{\nu}_{(0)}$$

- za struju više ne vrijedi zakon očuvanja $$0 = \partial_\mu\partial_\nu F^{\mu\nu} = -e\partial_\mu j^{\mu}_{(0)} \neq 0!$$

NEKONZISTENTNOST!

RAZLOG: $j^{\mu}_{(0)}$ nije struja vezana uz $\mathcal{L}_1$, već uz $\mathcal{L}_0$

očuvana struja je $j^{\mu}_{(1)} = j^{\mu}_{(0)} + eA_\mu\phi^*\phi$

Noetherina metoda

- općenito: iz $S_0 = \int d^4x \mathcal{L}_0$ invarijantne na transformacije $\delta_0\varphi$

konstruiramo

$$S = \int d^4x(\mathcal{L}_0 + \varepsilon\mathcal{L}_1 + \varepsilon^2\mathcal{L}_2 ...) \qquad \delta\varphi = \delta_0\varphi + \varepsilon\delta_1\varphi + \varepsilon^2\delta_2\varphi...$$

takve da je S invarijantna na $\delta\varphi$

- izjednačimo parametre $\delta A_\mu = \partial_\mu \Lambda,$ lokalne transformacije

transformacija $\delta\phi = ie\Lambda\phi$

- za slobodni lagranžijan $\delta\mathcal{L}_0 = -e\partial_\mu\Lambda j^\mu_{(0)} \neq 0$ dodamo $\mathcal{L} \sim eA_\mu j^\mu_{(0)}$

$$\delta\mathcal{L}_1 = -e\partial_\mu\Lambda j^\mu_{(0)} + e\partial_\mu\Lambda j^\mu_{(0)}$$

$$- e^2\partial_\mu\Lambda A^\mu\phi^*\phi \neq 0$$ dodamo $\mathcal{L} \sim \frac{1}{2}e^2 A_\mu A^\mu \phi^*\phi$

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + eA_\mu j^\mu_{(0)} + \frac{1}{2}e^2 A_\mu A^\mu\phi^*\phi.$$

$$\delta\mathcal{L}_1 = -e^2\partial_\mu\Lambda A^\mu\phi^*\phi + e^2\partial_\mu\Lambda A^\mu\phi^*\phi = 0$$

Dobili smo konzistentnu teoriju vezanu na materiju!

Slobodno spin-2 polje

- Graviton: masa=0, spin=2

→ opisan simetričnim tenzorom $h^{\mu\nu}$

- izvor gravitacije je masa → u SRTP izvor je $t^{\mu\nu}$

→ tražimo vezanje takvo da vrijedi $D^{\mu\nu}(h) = \chi t^{\mu\nu}_{\text{matter}}$

+ zakon očuvanja $\partial_\mu t^{\mu\nu}_{\text{matter}} = 0$ → **BAŽDARNI UVJET** $\partial_\mu D^{\mu\nu}(h) = 0$

- želimo naći linearnu teoriju → najopćenitiji lagranžijan:

$$\mathcal{L} = a\partial_\mu h_{\nu\rho}\partial^\mu h^{\nu\rho} + b\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} + c\partial_\mu h\partial_\rho h^{\mu\rho} + d\partial_\mu h\partial^\mu h$$

- jednadžba gibanja $$D^{\alpha\beta}(h) = 2a\partial^2 h^{\alpha\beta} + 2b\partial_\mu\partial^{(\alpha}h^{\beta)\mu} + c\partial^\alpha\partial^\beta h + c\eta^{\alpha\beta}\partial_\mu\partial_\lambda h^{\mu\lambda} + 2d\eta^{\alpha\beta}\partial^2 h = 0$$

+ baždarni uvjet → $a = -\frac{1}{2}b = \frac{1}{2}c = -d$

$$\mathcal{L} = \frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\mu h^{\nu\rho} - \frac{1}{2}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} + \frac{1}{2}\partial_\mu h \partial^\rho h^{\mu\rho} - \frac{1}{4}\partial_\mu h \partial^\mu h$$

FIERZ-PAULI LAGRANŽIJAN

Jednadžba gibanja:

$$D_{\mu\nu}(h) \equiv \partial^2 h_{\mu\nu} - 2\partial^\rho \partial_{(\mu} h_{\nu)\rho} + \eta_{\mu\nu}\partial_\alpha \partial_\beta h^{\alpha\beta} + \partial_\mu \partial_\nu h - \eta_{\mu\nu}\partial^2 h = 0$$

Baždarna invarijantnost:

$$\delta\mathcal{L} = \left[\frac{\delta\mathcal{L}}{\delta h_{\nu\rho}} - \partial_\mu \frac{\delta\mathcal{L}}{\delta(\partial_\mu h_{\nu\rho})}\right]\delta h_{\nu\rho} = D^{\nu\rho}\delta h_{\nu\rho} \sim \partial_\nu D^{\nu\rho}\epsilon_\rho$$

$$\delta h_{\mu\nu} = -\partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu$$

Problem nekonzistentnosti i Noetherina metoda

Vezanje na materiju

$$\mathcal{L}(h,\phi) = \mathcal{L}_{\mathrm{FP}} + \mathcal{L}_{\mathrm{matter}}(\phi) + \frac{1}{2}\chi h_{\mu\nu} t^{\mu\nu}_{\mathrm{matter}}(\phi)$$

$$D^{\mu\nu}(h) = \chi t^{\mu\nu}_{\mathrm{matter}} \xrightarrow{\partial_\mu D^{\mu\nu}(h) = 0} \boxed{\partial_\mu t^{\mu\nu}_{\mathrm{matter}}(\phi) = 0}$$

ZAKON OČUVANJA

Je li u skladu s jednadžbama gibanja?

Tenzor energije i impulsa: $t^{\mu\nu}_{\mathrm{matter}}(\phi) = -\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}\eta^{\mu\nu}(\partial\phi)^2$

$$\mathcal{L}(h,\phi) = \frac{1}{2}\left(\eta_{\mu\nu} - \chi\bar{h}_{\mu\nu}\right)\partial^\mu\phi\partial^\nu\phi + \mathcal{L}_{\mathrm{FP}}$$

$$\partial^2\phi = \chi\partial_\mu(\bar{h}^{\mu\nu}\partial_\nu\phi)$$

Pokrata: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$

$$\boxed{\partial_\mu t^{\mu\nu}_{\mathrm{matter}}(\phi) = -\chi\partial_\lambda(\bar{h}^{\lambda\rho}\partial_\rho\phi)\partial^\nu\phi}$$

NEKONZISTENTNOST!

RAZLOG: očuvan je ukupni tenzor energije i impulsa!

Newtonska granica

$$S = -M \int d\xi \frac{1}{\sqrt{\eta_{\alpha\beta}\dot{X}^\alpha \dot{X}^\beta}} \left(\eta_{\mu\nu} + \frac{1}{2}\chi h_{\mu\nu}(X)\right)\dot{X}^\mu \dot{X}^\nu, \quad \dot{X}^\mu = \frac{dX^\mu}{d\xi}$$

- tenzor energije i impulsa slobodne čestice: $t^{\mu\nu} = -M\delta^\mu_{\ 0}\delta^\nu_{\ 0}\delta^3(\vec{x})$

$\downarrow$ $D^{\mu\nu}(h) = \chi t^{\mu\nu}_{\text{matter}}$

jednadžbe gibanja

$$D^{00}(h) = -\chi M \delta^{(3)}(\vec{x})$$

$$D^{ij}(h) = 0$$

- koristimo de Donderovo baždarenje: $\partial^\mu \bar{h}_{\mu\nu} = 0 \longrightarrow D_{\mu\nu}(\bar{h}) = \partial^2 \bar{h}_{\mu\nu}$

$$\Rightarrow \quad h^{\mu\mu} = \frac{2}{\chi}\phi, \quad \phi = \frac{\chi^2 M}{16\pi|\vec{x}|}$$

$\longrightarrow$ možemo ga identificirati s Newtonskim potencijalom:

$$S = -m\int dt \left[\sqrt{1-v^2} + \frac{1}{\sqrt{1-v^2}}\left(1+v^2\right)\phi\right]$$

$$\approx \int dt \left(\frac{1}{2}mv^2 - m\phi + \frac{1}{8}mv^4 - \frac{3}{2}mv^2\phi + \ldots\right)$$

- model daje dobro slaganje s ogibanjem svjetlosti, 0.75 puta od opažene vrijednosti zakreta perihela Merkura

Noetherina metoda

- slobodni lagranžijan

$$\mathcal{L}_0 = \frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\mu h^{\nu\rho} - \frac{1}{2}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} + \frac{1}{2}\partial_\mu h \partial_\rho h^{\mu\rho} - \frac{1}{4}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi$$

- jednadžbe gibanja $D_{\mu\nu}(h) = 0, \quad \partial^2\phi = 0$
- transformacije polja

$$\delta_0 x^\mu = \chi\Sigma^\mu$$

$$\delta_0 h_{\mu\nu} = -\partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu - \chi\Sigma^\lambda \partial_\lambda h_{\mu\nu}$$

$$\delta_0 \phi = -\chi\Sigma^\mu \partial_\mu \phi$$

- kanonski tenzori en. i impulsa

$$t^{\mu\nu}_{\text{matter}}(\phi) = -\partial^\mu\phi\partial^\nu\phi + \frac{1}{2}\eta^{\mu\nu}(\partial\phi)^2$$

$$t_{\mu\nu}(h) = -\frac{1}{2}\partial_\mu h_{\beta\gamma}\partial_\nu h^{\beta\gamma} + \partial_\mu h^{\beta\gamma}\partial_\beta h_{\nu\gamma}$$
$$-\frac{1}{2}\partial_\mu h \partial^\rho h_{\nu\rho} - \frac{1}{2}\partial^\rho h \partial_\mu h_{\nu\rho} + \frac{1}{2}\partial_\mu h \partial_\nu h + \eta_{\mu\nu}\mathcal{L}_{\text{FP}}$$

→ tenzori su očuvani: $\partial^\mu t_{\mu\nu}(\phi) = -\partial^2\phi\partial_\nu\phi = 0$

$$\partial^\mu t_{\mu\nu}(h) = -\frac{1}{2}\partial_\nu h_{\lambda\rho} D^{\lambda\rho}(h) = 0$$

- tenzor $t_{\mu\nu}(\phi)$ nije izvor za polje $h^{\mu\nu}$ ➡ dodajemo član $\mathcal{L} \sim \chi h^{\mu\nu} t_{\mu\nu}(\phi)$

➡ $$\mathcal{L}_1 = \mathcal{L}_0 + \frac{1}{2}\chi h^{\mu\nu} t_{\mu\nu}(\phi)$$

- jednadžbe gibanja

$$D_{\mu\nu}(h) = \chi t_{\mu\nu}(\phi)$$

$$\partial_\mu \partial^\mu \phi = \chi \partial_\mu \left(h^{\mu\nu} \partial_\nu \phi - \frac{1}{2} h \partial^\mu \phi \right)$$

⬇

$$0 = \partial^\mu D_{\mu\nu} = \chi \partial^\mu t_{\mu\nu}(\phi) = -\chi \partial^2 \phi \partial_\nu \phi \neq 0!$$

NEKONZISTENTNOST!

RAZLOG: očuvana je ukupna energija sustava ⟶ gravitacija se mora vezati na samu sebe

$$\mathcal{L}_2 = \frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\mu h^{\nu\rho} - \frac{1}{2}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} + \frac{1}{2}\partial_\mu h \partial_\rho h^{\mu\rho} - \frac{1}{4}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\chi h^{\mu\nu} t_{\mu\nu}(\phi) + \frac{1}{2}\chi h^{\mu\nu} t_{\mu\nu}(h)$$

ne daje dobru jednadžbu gibanja: $D_{\mu\nu}(h) = \chi t_{\mu\nu}(\phi) + \chi t_{\mu\nu}(h)$

trebamo složeniji član samomeđudjelovanja $\mathcal{L}_{\text{corr}} = \frac{1}{2}\chi h^{\mu\nu}\mathcal{L}_{\mu\nu} \sim \chi h(\partial h)(\partial h)$

takav da

$$\mathcal{L}_3 = \frac{1}{4}\partial_\mu h_{\nu\rho}\partial^\mu h^{\nu\rho} - \frac{1}{2}\partial_\mu h_{\nu\rho}\partial^\nu h^{\mu\rho} + \frac{1}{2}\partial_\mu h \partial_\rho h^{\mu\rho} - \frac{1}{4}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2}\chi h^{\mu\nu} t_{\mu\nu}(\phi) + \frac{1}{2}\chi h^{\mu\nu}\mathcal{L}_{\mu\nu}$$

$$t_{\mu\nu}(h) = \mathcal{L}_{\mu\nu} - \partial_\sigma \left(h_{\rho\lambda} \frac{\delta \mathcal{L}^{\rho\lambda}}{\delta \partial_\sigma h^{\mu\nu}} \right)$$

$$D_{\mu\nu}(h) = \chi t_{\mu\nu}(\phi) + \chi t_{\mu\nu}(h)$$

$$\partial^\mu \left(t_{\mu\nu}(\phi) + t_{\mu\nu}(h) \right) = 0$$

- simetrije → tražimo teoriju invarijantnu na lokalnu verziju simetrija polja ϕ i $h^{\mu\nu}$

  → $\Sigma^{\mu} = \epsilon^{\mu}$ parametri lokalnih transformacija

  → zahtjev: baždarne transformacije generiraju istu algebru na ϕ i $h^{\mu\nu}$

  → $[\delta_1^{\epsilon_1}, \delta_1^{\epsilon_2}] = \delta_1^{\epsilon_3(\epsilon_1,\epsilon_2)}$

→ nove transformacije polja:

$$\delta_1 h_{\mu\nu} = -\partial_\mu \epsilon_\nu - \partial_\nu \epsilon_\mu - \chi \epsilon^\lambda \partial_\lambda h_{\mu\nu} - \chi \partial_\mu \epsilon^\lambda h_{\nu\lambda} - \chi \partial_\nu \epsilon^\lambda h_{\mu\lambda}$$

$$\delta_1 \phi = -\chi \epsilon^\mu \partial_\mu \phi$$

- najopćenitiji izraz za $\mathcal{L}_{\text{corr}}$:

$$\mathcal{L}_{\text{corr}} = \frac{1}{2}\chi h^{\mu\nu}\left(\alpha \partial_\mu h_{\rho\lambda} \partial_\nu h^{\rho\lambda} + \beta \partial_\mu h_{\rho\lambda} \partial^\rho h^\lambda_{\ \nu} + \ldots\right)$$

20 članova, 16 koeficijenata → određujemo ih iz baždarnog uvjeta:

$$\partial_\mu t^{\mu\nu}(h) = \gamma_{\rho\lambda}{}^{\nu} D^{\rho\lambda}(h),$$

$$\gamma_{\rho\lambda\nu} = \frac{1}{2}\left(\partial_\rho h_{\lambda\nu} + \partial_\lambda h_{\nu\rho} - \partial_\nu h_{\rho\lambda}\right)$$

$$\mathcal{L}_{\mathrm{corr}} = \frac{1}{2}\chi h^{\mu\nu}\Big(-\frac{1}{2}\partial_\mu h_{\rho\lambda}\partial_\nu h^{\rho\lambda} - \partial^\rho h_\mu{}^\lambda \partial_\rho h_{\lambda\nu}$$
$$+ \partial^\lambda h^\rho_{(\mu}\partial_\rho h_{\nu)\lambda} + 2\partial^\lambda h^\rho{}_{(\mu}\partial_{\nu)} h_{\rho\lambda}$$
$$- \partial_{(\mu} h_{\nu)\lambda}\partial^\lambda h - \partial^\lambda h_{\mu\nu}\partial^\rho h_{\rho\lambda}$$
$$- \partial^\lambda h_{\lambda(\mu}\partial_{\nu)} h + \partial_\lambda h_{\mu\nu}\partial^\lambda h$$
$$+ \frac{1}{2}\partial_\mu h \partial_\nu h + \eta_{\mu\nu}\mathcal{L}_{\mathrm{FP}}\Big).$$

$$D_{\mu\nu}(h) = \chi t_{\mu\nu}(\phi) + \chi t_{\mu\nu}(h)$$
$$\partial^2\phi = \chi\partial_\mu\left(h^{\mu\nu}\partial_\nu\phi - \frac{1}{2}h\partial^\mu\phi\right)$$
$$\partial^\mu\left(t_{\mu\nu}(\phi) + t_{\mu\nu}(h)\right) = 0$$

→ konzistentan rezultat u prvom redu u χ

→ dobro slaganje s opaženim vrijednostima zakreta perihela Merkura

Ali računanje viših redova u χ je komplicirano!

Deserov argument

$$\mathcal{L}_0 = \chi\varphi^{\mu\nu}\left(\partial_\rho\Gamma_{\mu\nu}{}^\rho - \partial_\mu\Gamma_{\nu\rho}{}^\rho\right) + \eta^{\mu\nu}\left(\Gamma_{\lambda\mu}{}^\rho\Gamma_{\rho\nu}{}^\lambda - \Gamma_{\lambda\rho}{}^\rho\Gamma_{\mu\nu}{}^\lambda\right)$$

→ transformacije polja:

$$\delta\varphi_{\mu\nu} = -2\partial_{(\mu}\epsilon_{\nu)} + \eta_{\mu\nu}\partial_\rho\epsilon^\rho$$

$$\delta\Gamma_{\mu\nu}{}^\rho = -\chi\partial_\mu\partial_\nu\epsilon^\rho$$

→ jednadžbe gibanja:

$$\partial_\rho\Gamma_{\mu\nu}{}^\rho - \partial_{(\mu}\Gamma_{\nu)\rho}{}^\rho = 0$$

$$\chi\partial_\rho\varphi^{\mu\nu} - \chi\partial_\lambda\varphi^{\lambda(\mu}\delta^{\nu)}_\rho - 2\Gamma_\rho{}^{(\mu\nu)} + \Gamma_\lambda{}^{\lambda(\mu}\delta^{\nu)}_\rho + \eta^{\mu\nu}\Gamma_{\rho\lambda}{}^\lambda = 0$$

→ djelujemo s δ^ρ_μ i $\eta_{\mu\nu}$ →

$$\Gamma_{\rho\lambda}{}^\lambda = -\frac{1}{2}\chi\partial_\rho\varphi, \quad \varphi = \varphi_\mu^\mu$$

$$\Gamma_\rho{}^{\rho\nu} = \chi\partial_\sigma\varphi^{\sigma\nu}$$

$$\Gamma_\rho{}^{(\mu\nu)} = \chi\partial_\rho h^{\mu\nu}, \quad h^{\mu\nu} = \varphi^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\varphi$$

→ $$\Gamma_{\rho\mu\nu} = \frac{1}{2}\chi\left(\partial_\rho h_{\mu\nu} + \partial_\mu h_{\nu\rho} - \partial_\nu h_{\rho\mu}\right)$$ → polja nisu nezavisna!

$$\Gamma_{\rho\mu\nu} = \frac{1}{2}\chi\left(\partial_\rho h_{\mu\nu} + \partial_\mu h_{\nu\rho} - \partial_\nu h_{\rho\mu}\right) \longrightarrow \partial_\rho \Gamma_{\mu\nu}{}^{\rho} - \partial_{(\mu}\Gamma_{\nu)\rho}{}^{\rho} = 0$$

$$\Rightarrow \boxed{-\frac{1}{2}\chi\left(D_{\mu\nu}(h) - \frac{1}{2}\eta_{\mu\nu}D_\rho{}^\rho(h)\right) = 0}$$

ista jednadžba gibanja kao za Fierz-Pauli lagranžijan!

- tražimo korekciju na $S_0 = \int d^4x \mathcal{L}_0$ zbog samointerakcije takvu da vrijedi $D_{\mu\nu}(h) = \chi t_{\mu\nu}$

$\longrightarrow$ koristimo Rosenfeldovu metodu:

1. $\eta_{\mu\nu} \longrightarrow \gamma_{\mu\nu}$
2. $\partial_\mu \longrightarrow \nabla_\mu$
3. $d^4x \longrightarrow \sqrt{|\gamma|}d^4x$
4. polja: tenzori ili gustoće tenzora?

$\varphi^{\mu\nu}$ je gustoća tenzora (transformira se kao $\sqrt{|\gamma|}f^{\mu\nu}$) !

$$\Rightarrow S_0 = \frac{1}{\chi^2}\int d^4x\Big[\chi\varphi^{\mu\nu}\Big(2\partial_{[\rho}\Gamma_{\mu]\nu}{}^{\rho} + 2C_{\nu[\mu}{}^{\sigma}\Gamma_{\rho]\sigma}{}^{\rho} - 2C_{\sigma[\mu}{}^{\rho}\Gamma_{\rho]\nu}{}^{\sigma}\Big) + \sqrt{|\gamma|}\gamma^{\mu\nu}2\Gamma_{\lambda[\mu}{}^{\rho}\Gamma_{\rho]\nu}{}^{\lambda}\Big]$$

$$t_{\mu\nu} = -\frac{2}{\sqrt{|\gamma|}} \frac{\delta S_0}{\delta \gamma^{\mu\nu}}\bigg|_{\gamma_{\mu\nu}=\eta_{\mu\nu}}$$

- ukupni lagranžijan: $\mathcal{L}_1 = \chi\varphi^{\mu\nu} 2\partial_{[\rho}\Gamma_{\mu]\nu}{}^{\rho} + (\eta^{\mu\nu} - \chi\varphi^{\mu\nu})\, 2\Gamma_{\lambda[\mu}{}^{\rho}\Gamma_{\rho]\nu}{}^{\lambda}$

  - korekcija nema član $\eta^{\mu\nu}$ koji se treba zamjeniti s $\gamma^{\mu\nu}$

  - nema novog doprinosa tenzoru en. i impulsa $t_{\mu\nu}$

- jednadžbe gibanja: $R_{\mu\nu}(\Gamma) = 0$ Riccijev tenzor

$$\begin{aligned} &-\chi\partial_\rho\varphi^{\mu\nu} + \chi\partial_\lambda\varphi^{\lambda(\mu}\delta^{\nu)}_{\rho} + 2\Gamma_{\rho}{}^{(\mu\nu)} \\ &\quad - \Gamma_{\lambda}{}^{\lambda(\mu}\delta^{\nu)}_{\rho} - \eta^{\mu\nu}\Gamma_{\rho\lambda}{}^{\lambda} - 2\chi\varphi^{\delta(\mu}\Gamma_{\rho\delta}{}^{\nu)} \\ &\quad + \chi\varphi^{\mu\nu}\Gamma_{\rho\sigma}{}^{\sigma} + \chi\varphi^{\lambda\sigma}\Gamma_{\lambda\sigma}{}^{(\mu}\delta^{\nu)}_{\rho} = 0 \end{aligned}$$

- definiramo:

$$\eta^{\mu\nu} - \chi\varphi^{\mu\nu} = g'^{\mu\nu}$$

$$g'_{\mu\nu} g'^{\nu\rho} = \delta_\mu^\rho$$

$$g_{\mu\nu} = \sqrt{|g'|}\, g'_{\mu\nu}$$

→ $g_{\mu\nu}$ je beskonačni red od $\varphi_{\mu\nu}$, ponaša se kao metrika

- na jednadžbu za ${\Gamma_{\mu\nu}}^{\rho}$ djelujemo s $g_{\mu}^{'\rho}$ i $g_{\mu\nu}^{'}$ + izraz za $g_{\mu\nu}$

$$\Gamma_{\rho\mu}{}^{\sigma} g_{\sigma\nu} + \Gamma_{\rho\nu}{}^{\sigma} g_{\sigma\mu} = \partial_\rho g_{\mu\nu}$$

$$\Gamma_{\rho\mu\nu} = \frac{1}{2}\left(\partial_\rho g_{\mu\nu} + \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu}\right)$$

$$\boxed{R_{\mu\nu}(\Gamma) = R_{\mu\nu}(g) = 0}$$

Einsteinova jednadžba

- konzistentnost: na jednadžbu za ${\Gamma_{\mu\nu}}^{\rho}$ djelujemo s δ_{μ}^{ρ} i $\eta_{\mu\nu}$

$$\begin{aligned}
\Gamma_{\rho\mu\nu} &= \frac{1}{2}\chi\left(\partial_\rho h_{\mu\nu} + \partial_\mu h_{\nu\rho} - \partial_\nu h_{\rho\mu}\right) \\
&\quad + \frac{1}{2}\left(f_{\rho\mu\nu} + f_{\mu\nu\rho} - f_{\nu\rho\mu}\right), \\
f_{\rho\mu\nu} &= 2\chi\varphi^{\delta}{}_{(\mu|}\Gamma_{\rho\delta|\nu)} - \chi\varphi_{\mu\nu}\Gamma_{\rho\delta}{}^{\delta} \\
&\quad - \frac{1}{2}\chi\eta_{\mu\nu}\left(2\varphi^{\delta}{}_{\lambda}\Gamma_{\rho\delta}{}^{\lambda} - \varphi\Gamma_{\rho\delta}{}^{\delta}\right)
\end{aligned}$$

$$R_{\mu\nu}(\Gamma) = \frac{1}{2}\chi\left(D_{\mu\nu}(h) - \frac{1}{2}\eta_{\mu\nu}D_{\rho}{}^{\rho}(h)\right) + 2\Gamma_{\lambda[\mu}{}^{\rho}\Gamma_{\rho]\nu}{}^{\lambda} - \frac{1}{2}\partial_{\tau}\Big[f_{\mu}{}^{\tau}{}_{\nu} + f_{\nu\mu}{}^{\tau} - f^{\tau}{}_{\nu\mu} + \chi\eta^{\tau}{}_{(\nu|}\left(2\varphi^{\delta}{}_{\lambda}\Gamma_{|\mu)\delta}{}^{\lambda} - \varphi\Gamma_{|\mu)\delta}{}^{\delta}\right)\Big]$$

► u skladu s prijašnjim izrazima

- za invarijantnost $\mathcal{L}_1$ na generalne transformacije koordinata

► dodajemo članove $\eta^{\mu\nu}\left[\partial_{\mu}\Gamma_{\nu\rho}{}^{\rho} - \partial_{\rho}\Gamma_{\mu\nu}{}^{\rho}\right]$:

$$\begin{aligned}\mathcal{L}_2 &= \left(\chi\varphi^{\mu\nu} - \eta^{\mu\nu}\right)2\partial_{[\rho}\Gamma_{\mu]\nu}{}^{\rho} + \left(\eta^{\mu\nu} - \chi\varphi^{\mu\nu}\right)2\Gamma_{\lambda[\mu}{}^{\rho}\Gamma_{\rho]\nu}{}^{\lambda} \\ &= g^{\mu\nu}\left(\partial_{\mu}\Gamma_{\nu\rho}{}^{\rho} - \partial_{\rho}\Gamma_{\mu\nu}{}^{\rho}\right) + g^{\mu\nu}\left(\Gamma_{\mu\lambda}{}^{\rho}\Gamma_{\nu\rho}{}^{\lambda} - \Gamma_{\lambda\rho}{}^{\rho}\Gamma_{\mu\nu}{}^{\lambda}\right) \\ &= g^{\mu\nu}R_{\mu\nu}(g)\end{aligned}$$

EINSTEIN-HILBERT LAGRANŽIJAN

Zaključak

- graviton mora biti bezmaseno polje spina 2 opisano simetričnim Lorentzovim tenzorom $h_{\mu\nu}$
- najopćenitiji lagranžijan za slobodno polje $h_{\mu\nu}$ je Fierz-Pauli lagranžijan
- problem nekonzistentnosti pri vezanju na materiju

1. rješenje: Noetherina metoda - dobri rezultati u prvom redu s obzirom na χ

  - komplicirana u višim redovima

2. rješenje: Deserov argument - konzistentna teorija

  - reproducira Hilbert-Einstein lagranžijan

LITERATURA

[1] R.P. Feynman, “Lectures on gravitation”, *edited by F. B. Morinigo, W. G. Wagner, and B. Hatifeld. 1995.*
[2] T. Ortin, “Gravity and strings”. Cambrige University Press, 2004.
[3] M. Fierz i W. E. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field.” *Proc. R. Soc. Lond.* (1939), str. 211.-232.
[4] V. I. Ogievetsky i I. V. Polubarinov, “Interacting field of spin 2 and the Einstein equations”. *Anals of Physics* 35.2 (1965), str. 167-208.
[5] R. M. Wald, “Spin-two fields and general covariance”. *Physical Review D.* 33.12 (1986), str. 3613.