

# Monte Carlo replike dubokog virtualnog komptonskog raspršenja

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# Uvod

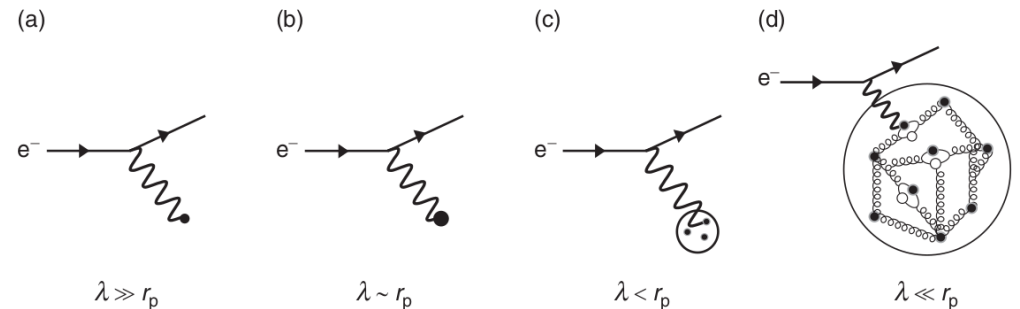
- ♦ Elektron-proton raspršenje se koristi kao alat za ispitivanje svojstava protona
- ♦ Fizika koje opisuje ovaj proces ovisi o energiji upadnog elektrona
  - veća energija elektrona => manja valna duljina virtualnog fotona
- ♦ Ovisno o odnosu valne duljine virtualnog fotona i radijusa protona, proces e-p raspršenja je moguće u grubo razdvojiti na četiri slučaja:

(a)  $\lambda \gg r_p$ : elastično raspršenje elektrona na točkastom protonu

(b)  $\lambda \sim r_p$ : aproksimacija točkastog protona više nije dobra, mora se uzeti u obzir i distribucija naboja i magnetskog momenta unutar protona

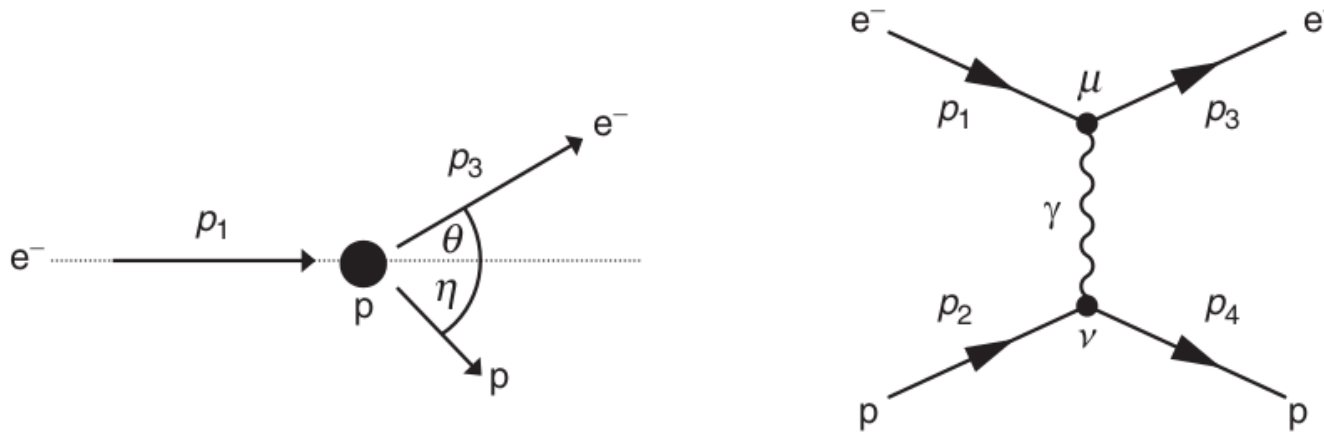
(c)  $\lambda > r_p$ : duboko neelastično raspršenje postaje dominantan proces

(d)  $\lambda \ll r_p$ : počinju se razlučivati gluoni i kvarkovi mora unutar protona



# Teorija elektron-proton raspršenja

# Elastično raspršenje



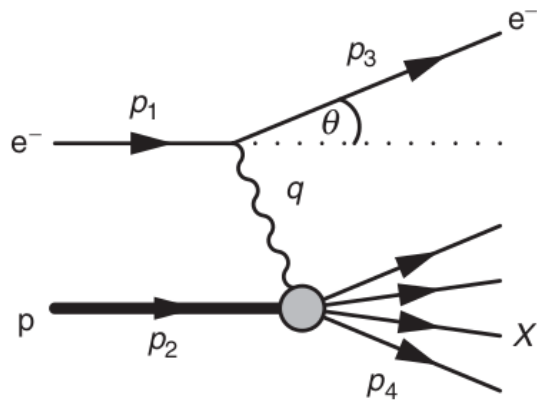
Najopćenitiji, Lorentz-invarijantni izraz za udarni presjek za elastično e-p raspršenje dan je tzv. **Rosenbluthovom formulom**:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2\left(\frac{\theta}{2}\right) + 2\tau G_M^2 \sin^2\left(\frac{\theta}{2}\right) \right]$$

$$G_E(Q^2) \approx G_E(\mathbf{q}^2) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3r$$

$$G_M(Q^2) \approx G_M(\mathbf{q}^2) = \int \mu(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3r$$

# Duboko neelastično raspršenje

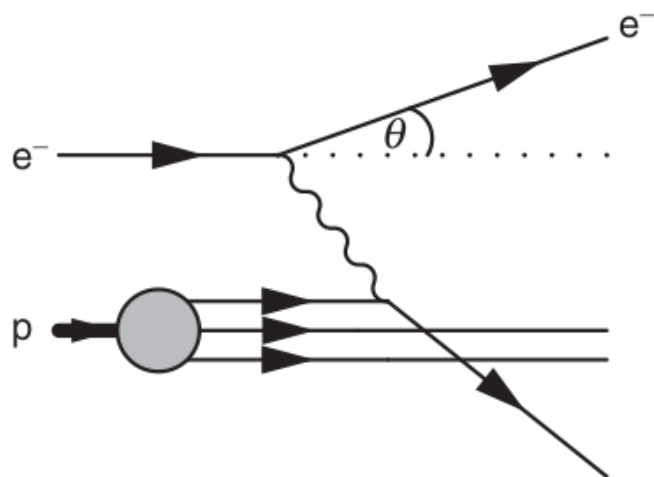


$$\frac{d^2\sigma}{dx_B d\Omega} = \frac{4\pi\alpha^2}{Q^4} \left[ (1-y) \frac{F_2(x_B, Q^2)}{x_B} + y^2 F_1(x_B, Q^2) \right]$$

Bjorkenovo skaliranje:  $F_j(x_B, Q^2) = F_j(x_B)$ ,  $j = 1, 2$

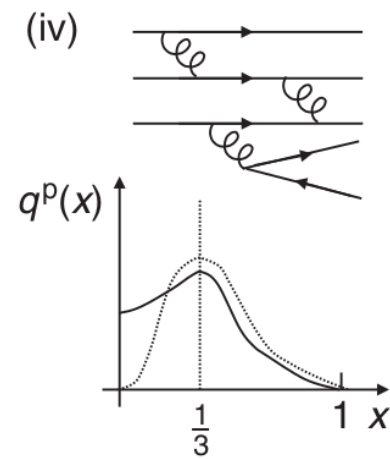
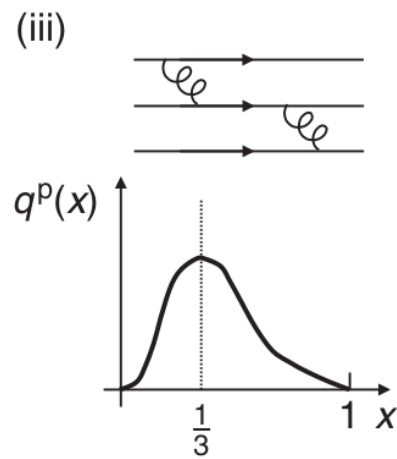
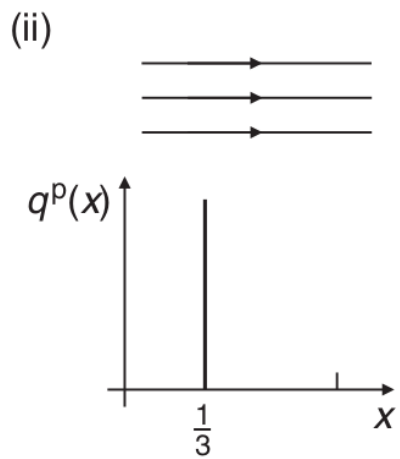
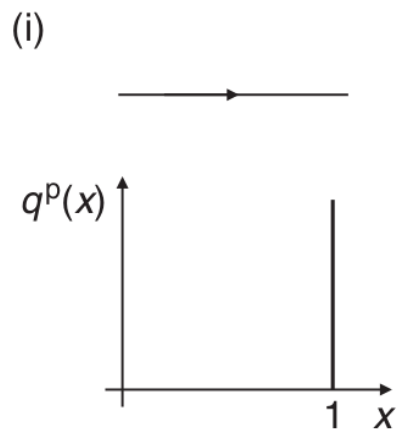
Callan-Gross relacija:  $F_2(x_B) = 2x_B F_1(x_B)$

# Kvark-partonski model



$$x = x_B$$

$$F_2(x) = 2xF_1(x) = x \sum_i Q_i^2 q_i^p(x)$$



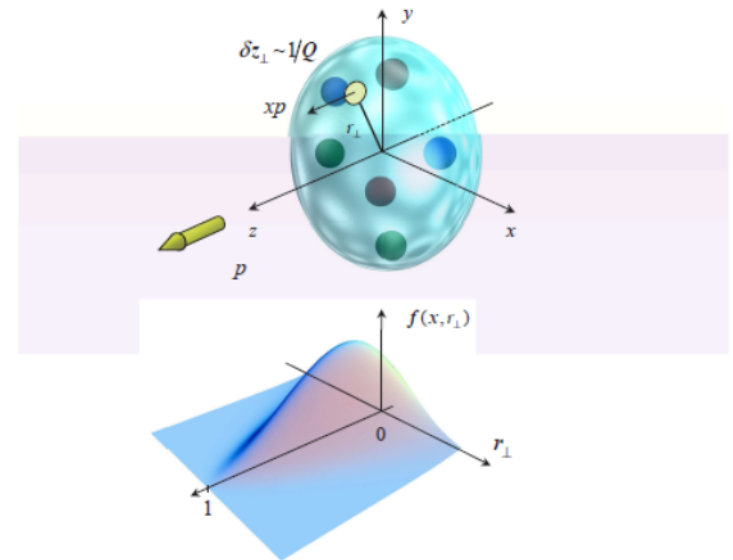
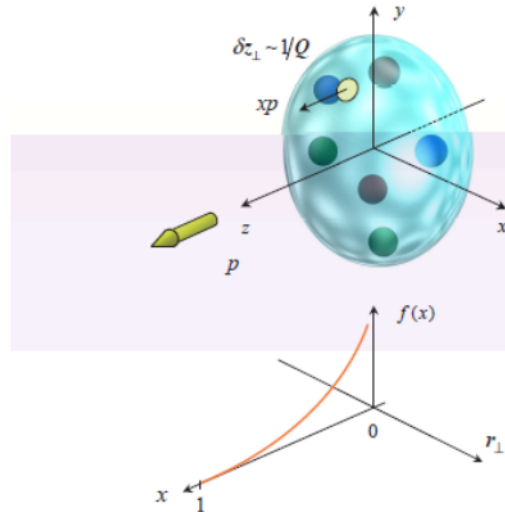
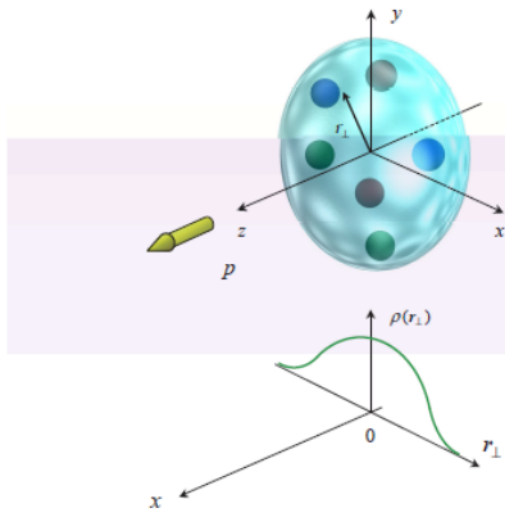
# Generalizirane partonske distribucije

- ♦ GPD-ovi efektivno daju 3D „sliku” obuhvaćanjem dvaju koncepata bitnih za razumijevanje strukture protona:
  - 1) Longitudinalne distribucije impulsa (PDF-ovi)
  - 2) Transverzalne distribucije naboja i mag. momenta (form faktori)
- ♦ Glavni i najpogodniji eksperiment za određivanje GPD-ova je tzv. **duboko virtualno komptonsko raspršenje** (*engl.* DVCS)
- ♦ Mjerenjem udarnog presjeka DVCS-a određuju se **komptonski form faktor** (*engl.* CFFs) koji su s GPD-ovima povezani preko konvolucija:

$$\mathcal{F}(\eta, t) = \sum_q e_q^2 \int_{-1}^{+1} \left[ \frac{1}{\eta - x - i\varepsilon} - \frac{1}{\eta + x - i\varepsilon} \right] F^q(x, \eta, t) dx$$

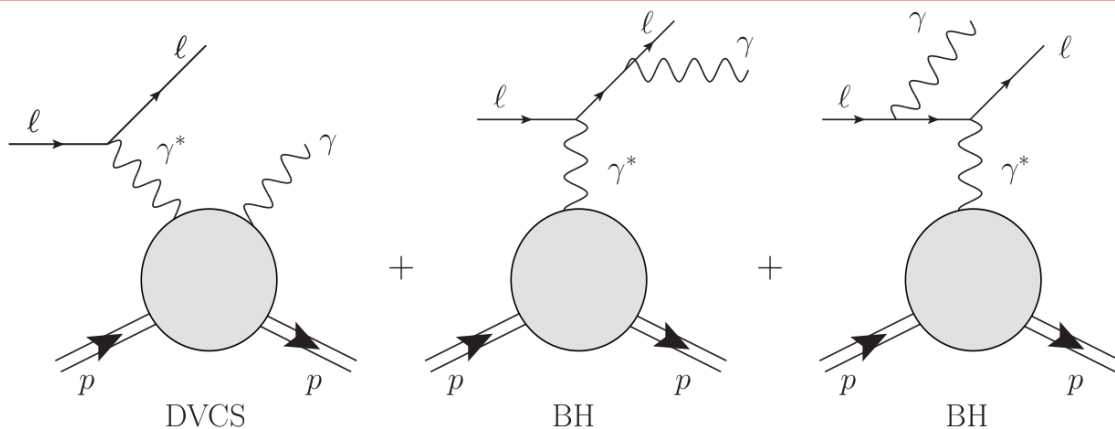
$$\tilde{\mathcal{F}}(\eta, t) = \sum_q e_q^2 \int_{-1}^{+1} \left[ \frac{1}{\eta - x - i\varepsilon} + \frac{1}{\eta + x - i\varepsilon} \right] \tilde{F}^q(x, \eta, t) dx$$

# Generalizirane partonske distribucije





# Duboko virtualno komptonско raspršenje



Pri mjerenju udarnih presjeka DVCS-a mora se uzeti u obzir i interferencija s tzv. **Bethe-Heitler** procesom

$$|\mathcal{T}|^2 = |\mathcal{T}_{\text{DVCS}}|^2 + |\mathcal{T}_{\text{BH}}|^2 + \mathcal{I}$$

$$|\mathcal{T}_{\text{DVCS}}|^2 = \frac{1}{Q^2} \sum_{n=0}^2 \left[ c_n^{\text{DVCS}} \cos(n\phi) + s_n^{\text{DVCS}} \sin(n\phi) \right]$$

$$|\mathcal{T}_{\text{BH}}|^2 = \frac{1}{x_B^2 t (1 + \varepsilon^2) \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left[ \sum_{n=0}^2 c_n^{\text{BH}} \cos(n\phi) + s_1^{\text{BH}} \sin(\phi) \right]$$

$$\mathcal{I} = \frac{-e_l}{x_B t y \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \sum_{n=0}^3 \left[ c_n^{\text{I}} \cos(n\phi) + s_n^{\text{I}} \sin(n\phi) \right]$$

# Tretman pogreške u mjerenjima

# Metoda najmanjih kvadrata

$$(\phi_n, y_n), n = 1, \dots, N \quad \longrightarrow \quad f(\phi) = \sum_{k=0}^{c_{max}} c_k \cos(k\phi) + \sum_{k=0}^{s_{max}} s_k \sin(k\phi)$$

Minimizacijom:  $S = \sum_{n=1}^N [y_n - f(\phi_n)]^2$

$$c_0 = \frac{1}{N} \sum_{n=1}^N y_n$$

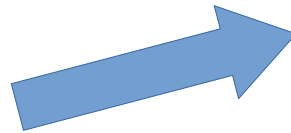
$$c_m = \frac{2}{N} \sum_{n=1}^N y_n \cos(m\phi_n)$$

$$s_m = \frac{2}{N} \sum_{n=1}^N y_n \sin(m\phi_n)$$

# Statistička pogreška

- ♦ Mjerenje je opterećeno isključivo statističkim (slučajnim) pogreškama
- ♦ Širina Gaussiana koji generira pogrešku je jednaka za svaku točku,  $\Delta y_n = \Delta$ , za svaki  $n$
- ♦ Greška za koeficijent  $c_m$  se određuje preko formule:

$$\Delta c_m = \sum_{n=1}^N \left( \frac{\partial c_m}{\partial y_n} \right)^2 \Delta y_n^2$$



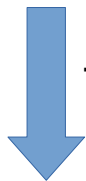
$$\Delta c_0 = \Delta \sqrt{\frac{1}{N}}$$

$$\Delta c_m = \Delta \sqrt{\frac{2}{N}}$$

$$\Delta s_m = \Delta \sqrt{\frac{2}{N}}$$

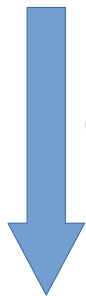
# Sistematska pogreška

$$\sigma_{c_m}^2 = \overline{(c_m - \bar{c}_m)^2}$$



Taylorov razvoj oko:  $c_m = \bar{c}_m$

$$\sigma_{c_m}^2 = \sum_{i,j=1}^N \frac{\overline{(y_i - \bar{y}_i)(y_j - \bar{y}_j)} \frac{\partial c_m}{\partial y_i} \frac{\partial c_m}{\partial y_j}}{\Big|_{c_m = \bar{c}_m}}$$



$$\text{cov}(y_i, y_j) = \overline{(y_i - \bar{y}_i)(y_j - \bar{y}_j)}$$

$$\sigma_{c_m}^2 = \frac{r_m^2}{N^2} \sum_{i,j=1}^N \text{cov}(y_i, y_j) \cos(m\phi_i) \cos(m\phi_j)$$

$\overline{(y_i - \bar{y}_i)} = \Delta \bar{y}_i$

$$\sigma_{c_m} = \bar{c}_m \Delta$$

$\overline{(y_i - \bar{y}_i)} = \Delta \cos(\phi_i) \bar{y}_i$

$$\sigma_{c_0} = \frac{\bar{c}_1}{2} \Delta$$

$$\sigma_{c_m} = \frac{\bar{c}_{m-1}}{r_{m-1}} \Delta$$

# Rezultati

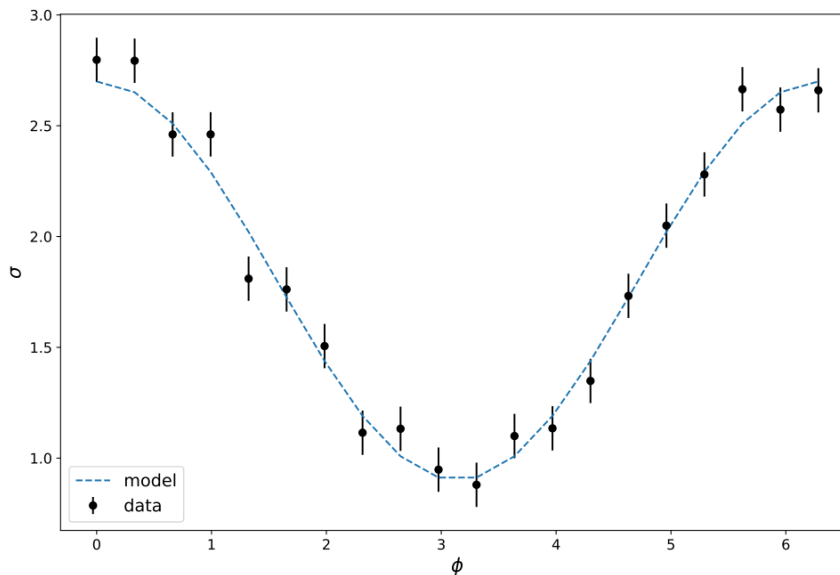
# Monte Carlo replike

$$\sigma(\phi) = c_0 + c_1 \cos(\phi)$$

$$\sigma_i = c_0^{\text{true}} + c_1^{\text{true}} \cos(\phi) + \Delta \cdot \varepsilon(\phi_i)$$

$$\sigma_i = \left( c_0^{\text{true}} + c_1^{\text{true}} \cos(\phi) \right) (1 + \Delta \cos(\phi_i) \cdot \varepsilon)$$

$$\sigma_i = \left( c_0^{\text{true}} + c_1^{\text{true}} \cos(\phi) \right) (1 + \Delta \cdot \varepsilon)$$

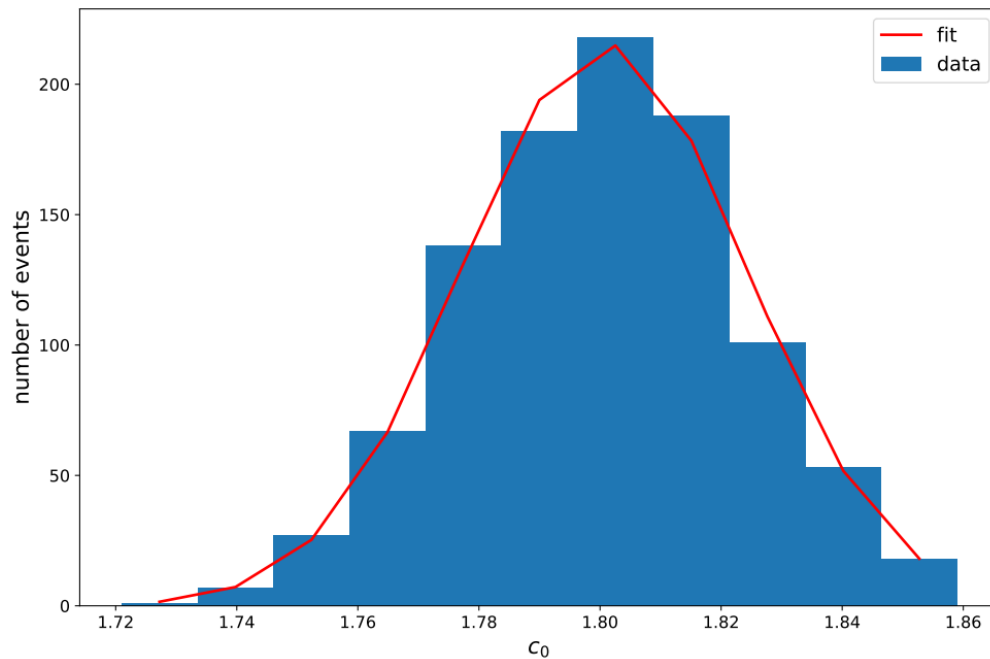


$$c_0^{\text{true}} = 1.8$$

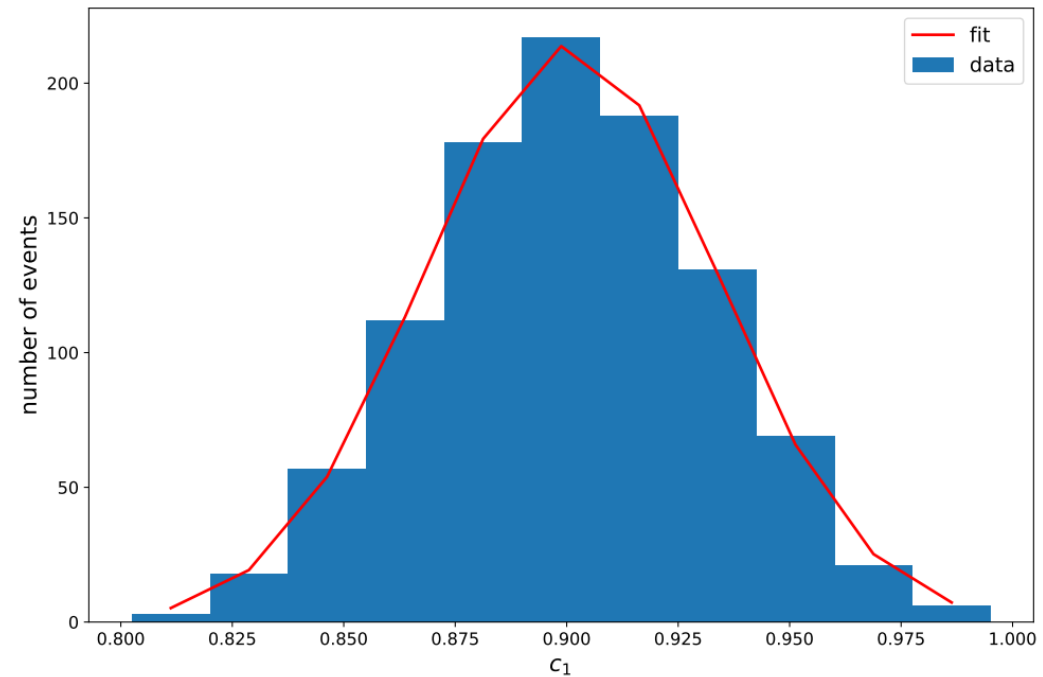
$$c_1^{\text{true}} = 0.9$$

$$\Delta = 0.1$$

# Statistička pogreška

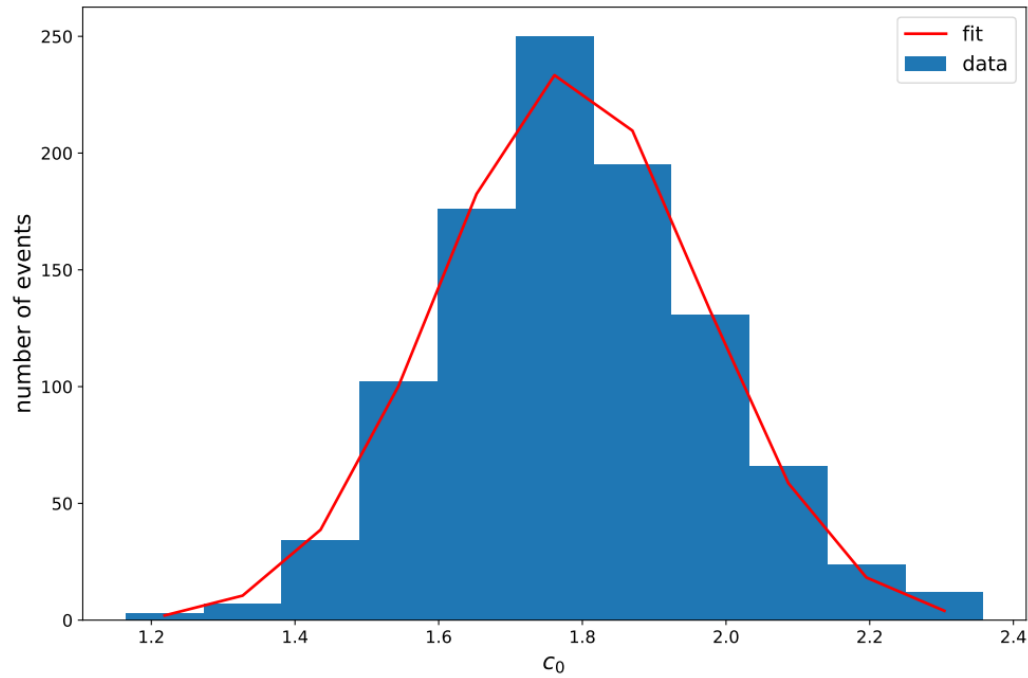


$$c_0 = 1.80 \pm 0.02 \quad \Delta c_0^{\text{th.}} \approx 0.02$$
$$c_1 = 0.90 \pm 0.03 \quad \Delta c_1^{\text{th.}} \approx 0.03$$

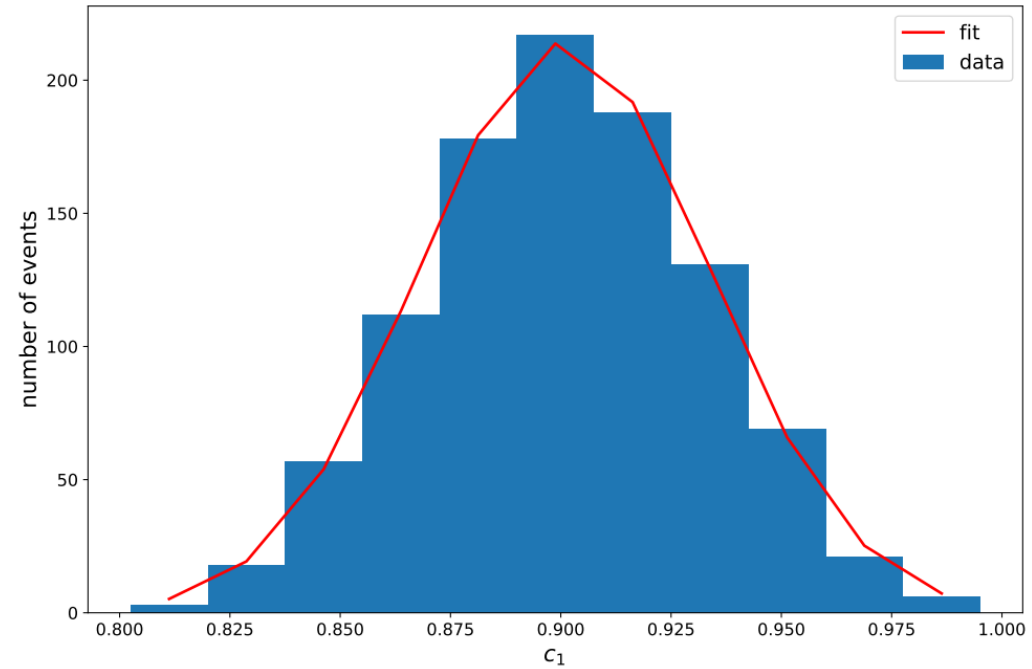




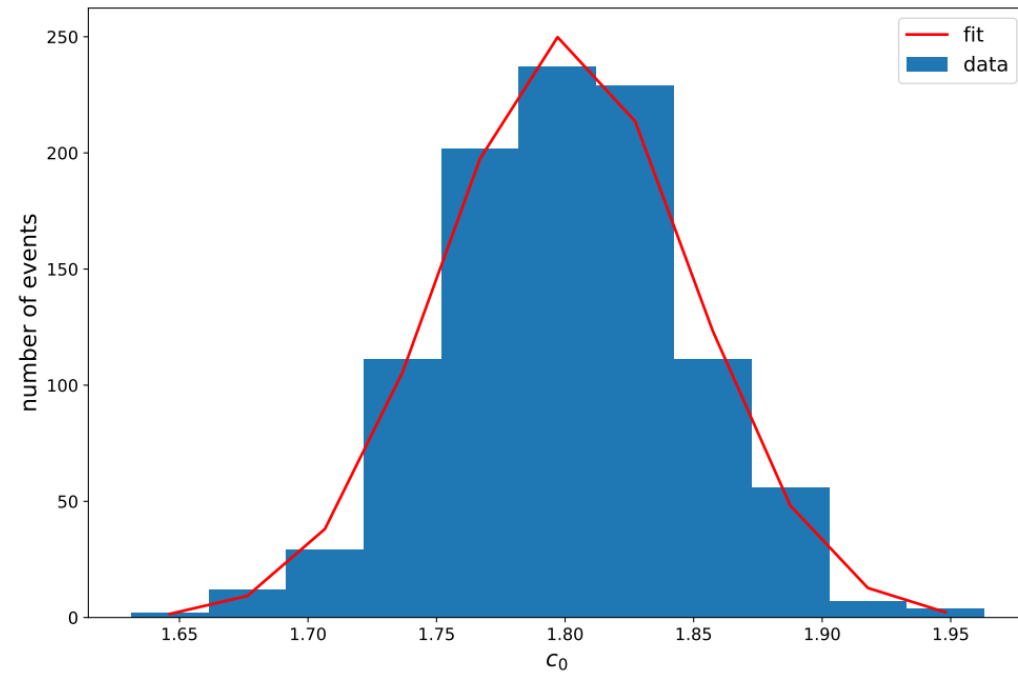
# Normalizacijska greška



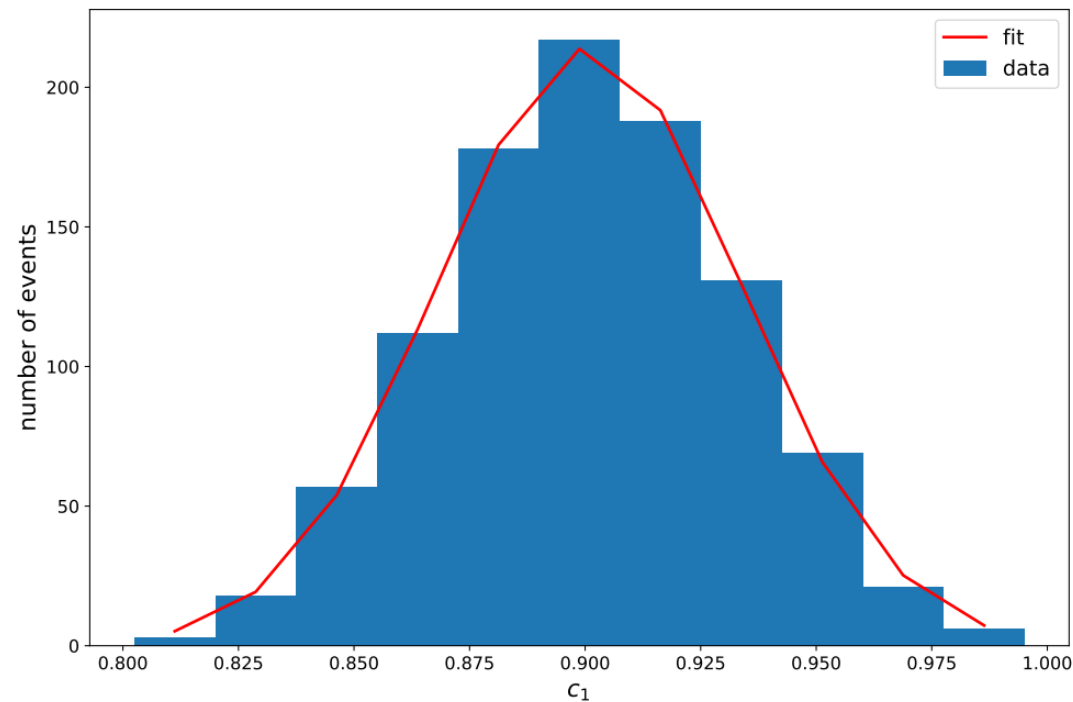
$$c_0 = 1.8 \pm 0.2 \quad \sigma_{c_0}^{\text{th.}} = 0.18$$
$$c_1 = 0.89 \pm 0.09 \quad \sigma_{c_1}^{\text{th.}} = 0.09$$



# Modulirana normalizacijska pogreška



$$c_0 = 1.80 \pm 0.05 \quad \sigma_{c_0}^{\text{th.}} = 0.045$$
$$c_1 = 0.9 \pm 0.2 \quad \sigma_{c_1}^{\text{th.}} = 0.18$$



# Zaključak

- ◆ Rezultati ukazuju da su Monte Carlo replike dobar oblik testiranja analitičkih izraza za nepouzdanosti parametara modela
- ◆ Preko poznatih izraza za nepouzdanosti parametara modela, propagacijom pogreške, se mogu odrediti nepouzdanosti komptonskih form faktora, a samim time i GPD-ova