Ograničenje skale prostorvremena

detekcijom gravitacijskih valova

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Sadržaj

Uvod
u problematiku

- "LIGO/Virgo Collaboration" : događaj **GW150914**
  - prva detekcija gravitacijskog vala
  - konstrikcija skale nekomutativnosti prostorvremena

Nekomutativno prostorvrijeme?

$$[\hat{x}^{\mu}, \hat{x}^{\nu}] = i\theta^{\mu\nu}$$

- Motivacija: generalizacija teorije polja na zakrivljeno prostorvrijeme
- Nekomutativnosti inducira skalu prostorvremena
  - Analogno reduciranoj Planckovoj konstanti ħ u faznom prostoru

Uvod
u opću teoriju relativnosti

Metrika $g_{\mu\nu}$

Riemannov tenzor zakrivljenosti:

$$R^{\sigma}{}_{\mu\alpha\beta} = \partial_\alpha \Gamma^{\sigma}_{\mu\beta} - \partial_\beta \Gamma^{\sigma}_{\mu\alpha} + \Gamma^{\sigma}_{\alpha\lambda}\Gamma^{\lambda}_{\mu\beta} - \Gamma^{\sigma}_{\beta\lambda}\Gamma^{\lambda}_{\mu\alpha}$$

Riccijev tenzor $R_{\mu\nu}$

Riccijev skalar R

Einsteinova jednadžba

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}}$$

Tenzor energije i impulsa $T_{\mu\nu}$

Gravitacijski valovi

u linearnoj aproksimaciji

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1$$

Pokrate: $h = \eta^{\mu\nu} h_{\mu\nu}$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$$

Loretzovo baždarenje $\partial^{\nu}\bar{h}_{\mu\nu} = 0$

U adekvatno odabranom koordinatnom sustavu:

$$\boxed{\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}}$$

d'Alembertian: $\Box = \partial^{\mu}\partial_{\mu} = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\boxed{\bar{h}_{\mu\nu}(t, \mathbf{x}) = \frac{4G}{c^4} \int d^3\mathbf{x}' \frac{1}{|\mathbf{x} - \mathbf{x}'|} T_{\mu\nu}\left(t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}, \mathbf{x}'\right)}$$

Gravitacijski valovi

Einsteinova kvadrupolna formula

Daleko od izvora: $|\mathbf{x}-\mathbf{x}'| \approx r - \mathbf{x}'\cdot\hat{\mathbf{n}}, \qquad |\mathbf{x}| = r, \quad \hat{\mathbf{x}} = \hat{\mathbf{n}}$

$$T_{\mu\nu}\left(t-\frac{r}{c}+\frac{\mathbf{x}'\cdot\hat{\mathbf{n}}}{c},\mathbf{x}'\right) = T_{\mu\nu}\left(t-\frac{r}{c},\mathbf{x}'\right) + \frac{x'^i n^i}{c}\partial_t T_{\mu\nu}\left(t-\frac{r}{c},\mathbf{x}'\right) + \\ + \frac{x'^i x'^j n^i n^j}{2c^2}\partial_t^2 T_{\mu\nu}\left(t-\frac{r}{c},\mathbf{x}'\right) + ...$$

Definiramo momente tenzora energije i impulsa

$$S_{kl,i_1...i_n} = \int d^3\mathbf{x} T_{kl}(t,\mathbf{x}) x_{i_1}...x_{i_n}$$

Perturbacija metrike poprima oblik

$$\bar{h}_{ij}(x) = \frac{4G}{c^4 r}\left[S_{ij} + \frac{n^m}{c}\dot{S}_{ij,m} + \frac{n^m n^p}{2c^2}\ddot{S}_{ij,mp} + ...\right]_{\mathrm{ret}}$$

Kvadrupolna aproksimacija

$$\left[\bar{h}_{ij}(x)\right]_{quad} = \frac{2G}{c^4 r}\ddot{M}_{ij}(t-r/c)$$

Moment gustoće energije $M_{ij} = \frac{1}{c^2}S_{00,ij} \implies S_{ij} = \frac{1}{2}\ddot{M}_{ij}$

Gravitacijski valovi

Einsteinova kvadrupolna formula

Transverzalno baždarenje iščezavajućeg traga (TT)

$$h^{0\mu} = 0, \qquad h^i_{\ i} = 0, \qquad \partial^j h_{ij} = 0$$

Maseni kvadrupolni moment $\rho(t, \mathbf{x}) = T^{00}/c^2$

$$Q^{ij} = M^{ij} - \frac{1}{3}\delta^{ij} M_{kk} = \int d^3x \rho(t,x)(x^i x^j - \frac{1}{3} r^2 \delta^{ij})$$

Kvadrupolna aproksimacija perturbacije metrike

$$\left[h^{TT}_{ij}(t,\mathbf{x})\right]_{quad} = \frac{1}{r}\frac{2G}{c^4}\ddot{Q}_{ij}(t - r/c)$$

Snaga kvadrupolnog zračenja $P_{quad} = \int d\Omega \frac{r^2 c^3}{32\pi G}\left\langle \dot{h}^{TT}_{ij}\dot{h}^{TT}_{ij}\right\rangle$

Einsteinova kvadrupolna formula

$$\boxed{P_{quad} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\right\rangle}$$

Post-Newtonovski (PN) razvoj

Gravitacijski dominiran sustav $(v/c)^2 \sim R_s/d, \quad R_s = 2Gm/c^2$

Sustavi slabog gravitacijskog polja $v/c \ll 1$

$\Longrightarrow v/c$ prirodan parametar razvoja

Frekvencija gravitacijskog zračenja povezana s frekvencijom kolektivnog gibanja mase $\omega \sim \omega_s \sim v/d$

Preko reducirane valne duljine: $\lambda = c/\omega \sim cd/v$

Razvoj neke funkcije retardiranog vremena:

$$F(t - r/c) = F(t) - \frac{r}{c}\dot{F}(t) + \frac{r^2}{2c^2}\ddot{F}(t) + ...$$

Vremenska derivacija daje faktor ω

Razvoj po potencijama r/λ

PN razvoj vrijedi u bliskoj zoni sustava $r \ll \lambda$

Post-Newtonovski (PN) razvoj

Razvoj metrike
$$g_{00} = -1 + \frac{2}{c^2}V + \mathcal{O}((v/c)^4)$$
$$g_{0i} = \mathcal{O}((v/c)^3)$$
$$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2}V\right) + \mathcal{O}((v/c)^4)$$

Retardirani potencijal
$$V(t,\mathbf{x}) = G\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\left(\frac{\partial}{c\partial t}\right)^k \int d^3\mathbf{x}'\,|\mathbf{x}-\mathbf{x}'|^{k-1}\,\rho(\mathbf{x}',t)$$

Red razvoja $(v/c)^{2n}$ označavamo kao red nPN

Nekomutativna gravitacija

Energija i impuls binarnog sustava crnih rupa

Tenzor energije i impulsa binarnog sustava u OTR

$$T^{\mu\nu}_{GR}(x) = m_1\gamma_1(t)v_1^\mu(t)v_1^\nu(t)\delta^3(\mathbf{x}-\mathbf{y}_1(t))) + 1 \leftrightarrow 2$$

Nekomutativnost koordinata $\Longrightarrow$ Moyalov produkt

$$f(x)\star g(x) = f(x)g(x) + \sum_{n=1}^{\infty}\left(\frac{i}{2}\right)^n \frac{1}{n!}\theta^{\alpha_1\beta_1}...\theta^{\alpha_n\beta_n}\partial_{\alpha_1}...\partial_{\alpha_n}f(x)\partial_{\beta_1}$$

Tenzor energije i impulsa masivnog skalarnog polja ϕ

$$T^{\mu\nu}_{NC} = \frac{1}{2}(\partial^\mu\phi\star\partial^\nu\phi + \partial^\nu\phi\star\partial^\mu\phi) - \frac{1}{2}\eta^{\mu\nu}(\partial_\rho\phi\star\partial^\rho\phi - m^2\phi\star\phi)$$

Nakon kvantizacije

$$T^{\mu\nu} = m_1\gamma_1 v_1^\mu v_1^\nu\delta^3(\mathbf{x}-\mathbf{y}(t)) + \frac{m_1^3G^2\Lambda^2}{8c^4}v_1^\mu v_1^\nu\theta^k\theta^l\partial_k\partial_l\delta^3(\mathbf{x}-\mathbf{y}(t)) + 1 \leftrightarrow 2$$

Pokrata: $\boxed{\Lambda\theta^i = \frac{\theta^{0i}}{l_P t_P}}$ $\Longrightarrow$ Skala nekomutativnosti Λ

Jednadžbe gibanja

reda 2PN

Kovarijantno očuvanje tenzora energije i impulsa

$$\nabla_\nu T^{\mu\nu} = 0 \iff \frac{dP}{dt} = F$$

Pokrate $\quad \mathbf{v} = \mathbf{v}_1 - \mathbf{v}_2 \qquad \mathbf{a} = \mathbf{a}_1 - \mathbf{a}_2$

$$M = m_1 + m_2, \qquad \mu = \frac{m_1 m_2}{M}, \qquad \nu = \frac{m_1 m_2}{M^2}$$

PP kružnu orbitu binarnog sustava $\mathbf{a}^{2PN}_{\text{kr.orb.}} = -\Omega^2 \mathbf{y}$

$$(\Omega^{2PN})^2 = \frac{GM}{r^3}\left[1 + (\nu - 3)\gamma \; + \left(6 + \frac{41}{4}\nu + \nu^2 - \frac{3}{8}(1-2\nu)\Lambda^2\right)\gamma^2\right]$$

uz parametar $\gamma = \dfrac{GM}{c^2 r}$

$$E^{2PN}_{\text{kr.orb.}} = -\frac{\mu c^2 x}{2}\left[1+\left(-\frac{3}{4} - \frac{1}{12}\nu\right)x+\left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{1}{24}\nu^2 + \frac{1}{4}(1-2\nu)\Lambda^2\right)x^2\right]$$

uz tzv. frekvencijski parametar $x = \left(\dfrac{GM\Omega}{c^3}\right)^{2/3}$

Jednadžbe gibanja

gubitak energije binarnog sustava

Očuvanje energije $\frac{dE}{dt} = -\mathcal{F}, \qquad \mathcal{F} = \mathcal{F}_{tr} + \mathcal{F}_{rez}$

Einsteinova kvadrupolna formula $\mathcal{F}_{tr}^{2PN} = \frac{G}{5c^5} \dddot{Q}_{ij} \dddot{Q}_{ij}$

$$\mathcal{F}^{2PN} = \frac{32c^2}{5G}\nu^2 x^5 \Bigg[1 - \left(\frac{1247}{336} + \frac{35}{12}\nu\right) x + 4\pi x^{3/2} + \\ + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2 - \frac{1}{2}\Lambda^2(1-2\nu)\right) x^2\Bigg]$$

Orbitalna faza binarnog sustava

$$\phi^{2PN}(x) = -\frac{x^{-5/2}}{32\nu}\Bigg[1 + \left(\frac{3715}{1008} + \frac{55}{12}\nu\right) x - 10\pi x^{3/2} + \\ + \left(\frac{15293365}{1016064} + \frac{27145}{1008}\nu + \frac{3085}{144}\nu^2 + \frac{25}{4}(1-2\nu)\Lambda^2\right) x^2\Bigg]$$

Skala nekomutativnosti

Fazni predlošci

PP oblik signala gravitacijskog vala

$$h(t) = 2A(t)\cos(\Phi(t)) = A(t)\left[e^{-i\Phi(t)} + e^{i\Phi(t)}\right]$$

Fourierov transformat je oblika $\tilde{h}^{SPA}(f) = \sqrt{\frac{2\pi}{\ddot{\Phi}(t_f)}}A(t_f)e^{i\psi(f)}$

uz pokratu $\psi(f) = 2\pi f t_f - \pi/4 - \Phi(t_f)$

Faza gravitacijskog vala dvostruko je veća od orbitalne faze binarnog sustava $\dot{\Phi}(t) = 2\Omega(t) \implies \Phi(t) = 2\phi(t)$

Konačni izraz reda 2PN

$$\boxed{\psi^{2PN}(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\nu}\sum_{j=0}^{4}\varphi_j\left(\frac{\pi M G f}{c^3}\right)^{\frac{j-5}{3}}}$$

$$\varphi_0 = 1, \qquad \varphi_1 = 0,$$

$$\varphi_2 = \frac{3715}{756} + \frac{55}{9}\nu, \qquad \varphi_3 = -16\pi,$$

$$\varphi_4 = \frac{15293365}{508032} + \frac{27145}{504}\nu + \frac{3085}{72}\nu^2 + \frac{25}{2}(1-2\nu)\Lambda^2$$

Skala nekomutativnosti

Ograničenje

Odmak od OTR $\varphi_j \to \varphi_j(1+\delta\varphi_j)$

$$\delta\varphi_4^{NC} = \frac{\varphi_4^{NC}}{\varphi_4^{GR}} = \frac{1270080(1-2\nu)}{4353552\nu^2 + 5472432\nu + 3058673}\Lambda^2$$

LIGO/Virgo analiza:

$$\delta\varphi_4 = -1.9^{+19.3}_{-16.4} \implies \left|\delta\varphi_4^{NC}\right| \lesssim 20$$

$$\boxed{\delta\varphi_4^{NC} = 0.137\Lambda^2 \implies \Lambda \lesssim 12}$$

podsjetnik: $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$ $\quad \Lambda\theta^i = \dfrac{\theta^{0i}}{l_P t_P}$

Zaključak

- Nekomutativno prostorvrijeme dozvoljava poopćenje SM na zakrivljeno prostorvrijeme
- Vremenske komponente tenzora nekomutativnosti ograničene su na red veličine iznad Planckove skale
- Preostale komponente zahtijevaju više redove PN razvoja, koji uključuju povratne interakcije gravitacijskih valova sa izvorom zračenja

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