Asimptotska sloboda u ne-Abelovim teorijama

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24. Siječanj 2023.

Uvod

Definicija ne-Abelove teorije: teorija (lagranžijan) koji posjeduje svojstvo invarijantnosti na istovremenu lokalnu U(N) ili SU(N) transformaciju fermionskog polja:

$$\psi(x) \to \psi'(x) = e^{i\alpha_a(x)t^a}\psi(x) = U(x)\psi(x) \tag{1}$$

i baždarnu transformaciju gluonskog polja:

$$G'^{a}_{\mu}t^a = U(x)\left(G^a_\mu t^a + \frac{i}{g}\partial_\mu\right)U^\dagger(x). \tag{2}$$

Motivacija: lagranžijan kvantne elektrodinamike (QED) zadovoljava traženo svojstvo s obzirom na lokalne U(1) transformacije.

$$\mathcal{L}_{QED} = \bar{\psi}(i\not{\partial} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\bar{\psi}\not{A}\psi, \tag{3}$$

s definicijom elektromagnetskog (EM) tenzora:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4}$$

Yang i Mills [1] te Faddeev i Popov [4] su dobili rezultat koji predstavlja izravno poopćenje teorije (3):

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu} + g\bar{\psi}G\!\!\!/^{a}t^{a}\psi - \frac{1}{2\eta}(\partial_{\mu}G^{\mu a})^{2} + \bar{c}^{a}(-\delta^{ac}\partial_{\mu}\partial^{\mu} - gf^{abc}\partial_{\mu}G^{\mu b})c^{c}, \tag{5}$$

s definicijom gluonskog tenzora:

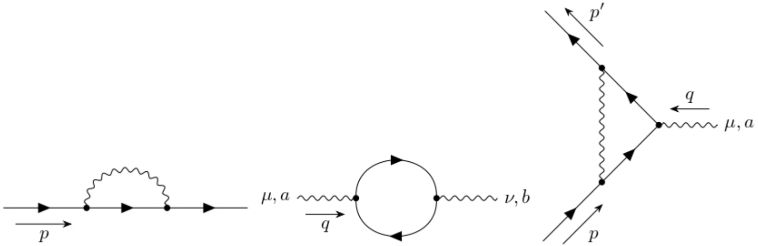
$$F^{a}_{\mu\nu} = \partial_{\mu}G^{a}_{\nu} - \partial_{\nu}G^{a}_{\mu} + gf^{abc}G^{b}_{\mu}G^{c}_{\nu}. \tag{6}$$

- strukturu ne-Abelove teorije komplicira opća nekomutativnost generatora grupa (oznaka: t^a) U(N) ili SU(N):

$$[t^{a}, t^{b}] = if^{abc}t^{c}$$

- pojava fermion-gluon-fermion, 3-gluonskog, 4-gluonskog te duh-gluon-duh vrha

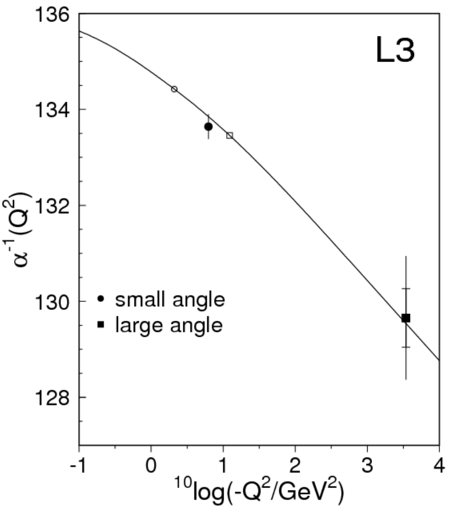
Landauov pol u kvantnoj elektrodinamici



- analiza kvantne elektrodinamike na razini jedne petlje $\rightarrow$ divergencije svih triju dijagrama
- sustavna procedura uklanjanja divergencija (renormalizacija) $\rightarrow$ klizanje konstante vezanja fermionskog i fotonskog polja $\alpha(k^2)$

$$\alpha(k^2) = \frac{\alpha_0^2}{1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{k^2}{M^2}\right)} \tag{7}$$

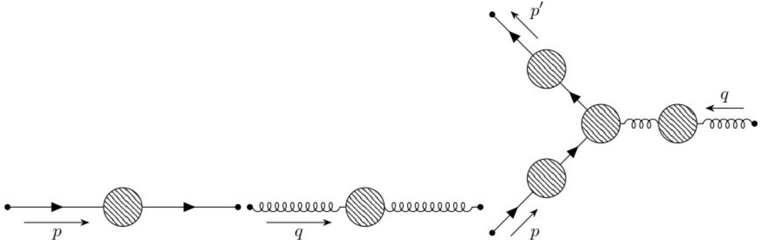
- $\alpha_0 \approx \frac{1}{137}, M = m_e$
- analiza ne-Abelove teorije (5) na razini jedne petlje i poopćenje izraza (7)



L3
small angle
large angle
136
134
132
130
128
-1
0
1
2
3
4
$\alpha^{-1}(Q^2)$
$^{10}\log(-Q^2/\mathrm{GeV}^2)$

Korelacijske funkcije

- fizikalna interpretacija 2-točkastih korelacijskih funkcija: amplituda širenja čestica iz točke x u točku y
- $\widetilde{G}^{(2,0)}(p)$, $\widetilde{G}^{(0,2)}(q)$, $\widetilde{G}^{(2,1)}(p, p', q)$



Definicije:

$$
\begin{aligned}
G^{(2,0)}(x,y)_{\alpha\beta} &= \langle\Omega| T\{\psi_\alpha(x)\bar{\psi}_\beta(y)\}|\Omega\rangle \\
&= \langle\Omega|\theta(x^0-y^0)\psi_\alpha(x)\bar{\psi}_\beta(y) \\
&- \theta(y^0-x^0)\bar{\psi}_\beta(y)\psi_\alpha(x)|\Omega\rangle,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
G^{ab\ (0,2)}_{\mu\nu}(x,y) &= \langle\Omega| T\{G^a_\mu(x)G^b_\nu(y)\}|\Omega\rangle \\
&= \langle\Omega|\theta(x^0-y^0)G^a_\mu(x)G^b_\nu(y) \\
&+ \theta(y^0-x^0)G^b_\nu(y)G^a_\mu(x)|\Omega\rangle.
\end{aligned} \tag{9}
$$

U slobodnim teorijama bez međudjelovanja:

$$\widetilde{G}_{sl}^{(2,0)}(p) = i\frac{\not{p} + m}{p^2 - m^2}, \tag{10}$$

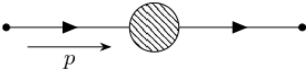
$$\widetilde{G}_{\mu\nu,sl}^{ab\ (0,2)}(q) = -\frac{i}{q^2}\left(g_{\mu\nu} - (1-\eta)\frac{q_\mu q_\nu}{q^2}\right)\delta^{ab}. \tag{11}$$

Källén–Lehmannova spektralna reprezentacija korelacijskih funkcija

U P i CPT invarijantnim teorijama vrijedi za 2-točkastu fermionsku korelacijsku funkciju ([2], [6]):

$$\widetilde{G}^{(2,0)}(p) \approx iZ_1 \frac{\not{p} + \widetilde{m}}{p^2 - \widetilde{m}^2}, \tag{12}$$

$$\widetilde{m} \neq m.$$



Reskaliranje (ili renormalizacija) polja ne-Abelove teorije $\psi(x)$, $G^{\mu a}(x)$ i $c^a(x)$:

$$\psi_r(x) = \frac{\psi(x)}{\sqrt{Z_1}}, \tag{13}$$

$$G_r^{\mu a}(x) = \frac{G^{\mu a}(x)}{\sqrt{Z_2}}, \tag{14}$$

$$c_r^a(x) = \frac{c^a(x)}{\sqrt{Z_3}}, \tag{15}$$

pretvara Källén–Lehmannovu korelacijsku funkciju u slobodni propagator (s fizikalnim masama).

Pogodnije je izraziti lagranžijan preko fizikalno mjerljivih polja i parametara:

$$\left(\psi_0(x), G_0^{\mu a}(x), c_0^a(x), m_0, g_0\right) \rightarrow \left(\psi(x), G^{\mu a}(x), c^a(x), m, g\right),$$

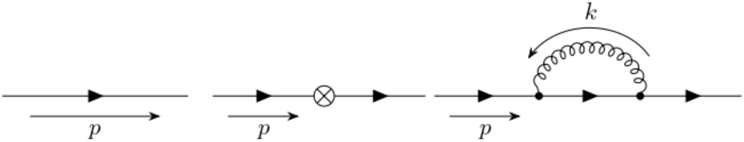
$$\mathcal{L} = \mathcal{L}_0 = \mathcal{L}_{ren} + \left(\mathcal{L}_0 - \mathcal{L}_{ren}\right), \tag{16}$$

$$\begin{aligned} \mathcal{L}_{ren} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} + g\bar{\psi}G\!\!\!/^a t^a \psi \\ - \frac{1}{2\eta}(\partial_\mu G^{\mu a})^2 + \bar{c}^a(-\delta^{ac}\partial_\mu\partial^\mu - gf^{abc}\partial_\mu G^{\mu b})c^c. \end{aligned} \tag{17}$$

$\mathcal{L}_0 - \mathcal{L}_{ren} \rightarrow$ kontračlanovi, uvode nova međudjelovanja u teoriju.

Račun 2-točkaste fermionske korelacijske funkcije $\widetilde{G}^{(2,0)}(p)$

- perturbativni razvoj: granasti dijagrami i dijagrami s jednom petljom
- tri dijagrama:

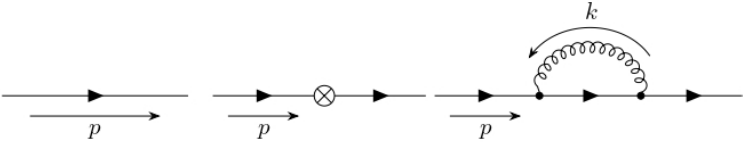


- treći dijagram divergira $\rightarrow$ regulariziramo amplitudu dijagrama dimenzijskom regularizacijom (Hooft, G. 't; Veltman, M.; [7])

Konačan rezultat:

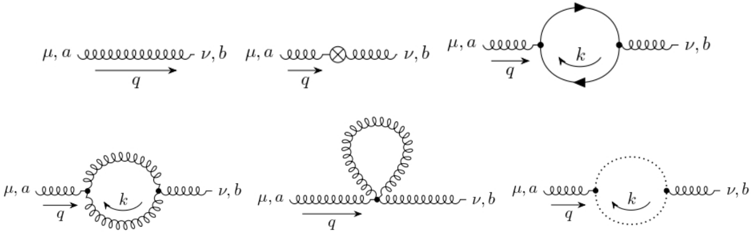
$$\widetilde{G}^{(2,0)}(p) = i\frac{\not{p}}{p^2} - \tag{18}$$

$$- i\frac{\not{p}}{p^2}\left(\delta_1 + g^2\frac{\Gamma(2-\frac{d}{2})}{(4\pi)^2}C_2(r)\frac{1}{(-p^2)^{2-\frac{d}{2}}}\right).$$



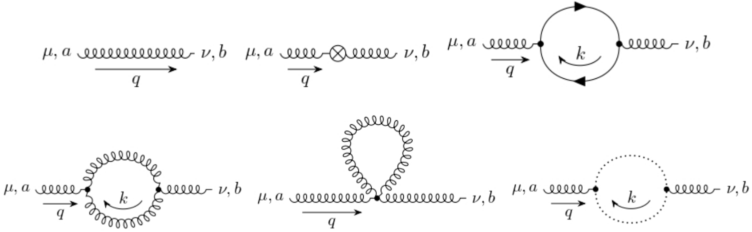
Račun 2-točkaste gluonske korelacijske funkcije $\widetilde{G}^{(0,2)}(q)$

- šest dijagrama:



Konačan rezultat:

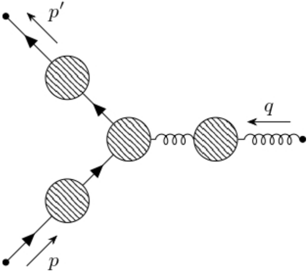
$$\widetilde{G}^{ab\ (0,2)}_{\mu\nu}(q) = \tag{19}$$
$$= -\frac{i}{q^2}\delta^{ab}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \frac{i}{q^2}\delta^{ab}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\cdot$$
$$\cdot\left(\delta_2 - \frac{g^2}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{(-q^2)^{2-\frac{d}{2}}}\left(\frac{5}{3}C_2(G) - \frac{4}{3}C(r)n_f\right)\right).$$



Račun 3-točkaste korelacijske funkcije $\widetilde{G}^{(2,1)}(p, p', q)$

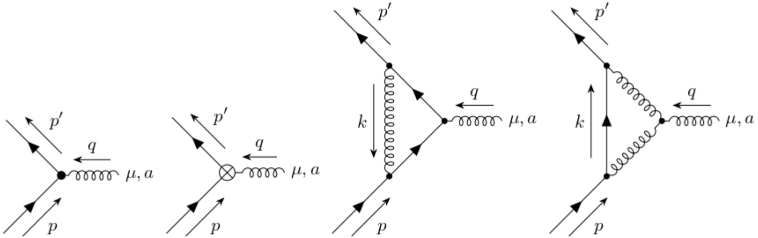
Faktorizacija:

$$\widetilde{G}_{\mu}^{a\ (2,1)}(p, p', q) = \widetilde{G}^{(2,0)}(p) C^{\nu b} \widetilde{G}^{(2,0)}(p') \widetilde{G}_{\nu\mu}^{ba\ (0,2)}(q). \qquad (20)$$



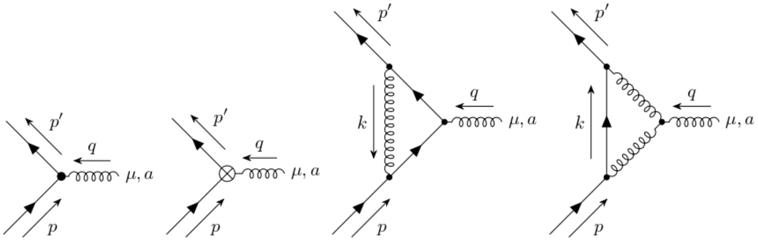
Račun fermion-gluon-fermion vrha $C^{\mu a}$

- četiri dijagrama:



Konačan rezultat:

$$\mathcal{C}^{\mu a} = ig\gamma^{\mu}t^{a} + \\ + i\gamma^{\mu}t^{a}\left(\delta_{fgf} + \frac{g^3}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{\Delta^{2-\frac{d}{2}}}\left(C_2(r) + C_2(G)\right)\right). \tag{21}$$



Renormalizacijski uvjeti

- izrazi (18), (19) i (21) moraju biti konačni (pojava u dobro definiranim opservabilnim veličinama poput udarnog presjeka)

Renormalizacijski uvjeti:

$$\widetilde{G}^{(2,0)}(p;\ p^2 = -M^2) = i\frac{\not{p}}{p^2}, \tag{22}$$

$$\widetilde{G}^{ab\ (0,2)}_{\mu\nu}(q;\ q^2 = -M^2) = -i\frac{\delta^{ab}}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right), \tag{23}$$

$$\mathcal{C}^{\mu a}(\Delta = M^2) = ig\gamma^\mu t^a. \tag{24}$$

Fiksiranje vrijednosti kontračlanova δ_1, δ_2 i δ_{fgf}.

$$\delta_1 = -\frac{g^2}{(4\pi)^2} C_2(r) \frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}, \tag{25}$$

$$\delta_2 = \frac{g^2}{(4\pi)^2} \left(\frac{5}{3} C_2(G) - \frac{4}{3} C(r) n_f \right) \frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}},$$

$$\delta_{fgf} = -\frac{g^3}{(4\pi)^2} (C_2(r) + C_2(G)) \frac{\Gamma(2-\frac{d}{2})}{(M^2)^{2-\frac{d}{2}}}.$$

- teorija ne može ovisiti o renormalizacijskoj skali $M \to$ klizanje konstante vezanja g u ovisnosti o skali M

Callan-Symanzikova jednadžba

$$\left(M\frac{\partial}{\partial M}+\beta(g)\frac{\partial}{\partial g}+n\gamma_1(g)+m\gamma_2(g)\right)\widetilde{G}^{(n,m)}=0, \qquad (26)$$

$$\rightarrow \beta(g)=M\frac{\partial g}{\partial M}=-\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_2(G)-\frac{4}{3}C(r)n_f\right). \qquad (27)$$

β funkcija ne-Abelove teorije

Rješenje diferencijalne jednadžbe (27):

$$g^2(k^2) = \frac{g_0^2}{1 + \frac{g_0^2}{(4\pi)^2}\left(\frac{11}{3}C_2(G) - \frac{4}{3}C(r)n_f\right)\ln\left(\frac{k^2}{M^2}\right)}, \tag{28}$$

Specijalni slučajevi

β funkcija kvantne elektrodinamike (QED): $C_2(G) = 0,\ C(r) = 1$

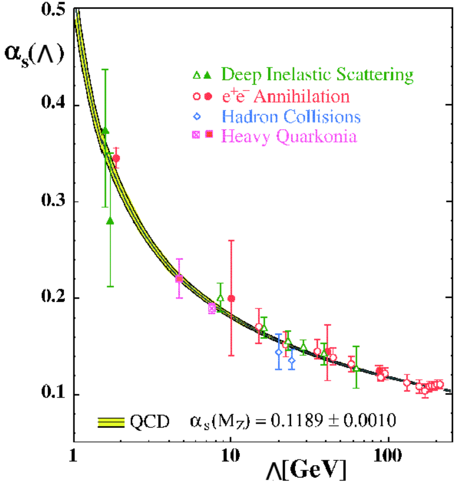
$$\alpha_{QED}(k^2) = \frac{\alpha_{0,QED}}{1 - \frac{\alpha_{0,QED}}{3\pi} \ln\left(\frac{k^2}{M^2}\right)}. \tag{29}$$

β funkcija teorija SU(N) grupe: $C_2(G) = N\ , C(r) = \frac{1}{2}$

$$g^2(k^2) = \frac{g_0^2}{1 + \frac{g_0^2}{(4\pi)^2}\left(\frac{11}{3}N - \frac{2}{3}n_f\right) \ln\left(\frac{k^2}{M^2}\right)}. \tag{30}$$

$\rightarrow$ asimptotska sloboda ukoliko $n_f < \frac{11}{2}N$
(općenitije $n_f < \frac{11}{4}\frac{C_2(G)}{C(r)}$)

Specijalan slučaj: kvantna kromodinamika



Zaključak

- mogućnost asimptotske slobode u ne-Abelovim teorijama
- postojanje Landauovog pola ili asimptotske slobode nije znak matematičke nekonzistentnosti teorije već neprimjenjivosti računa smetnje na nekim energijskim skalama
  - u slučaju QED-a, viši redovi računa smetnje pomiču Landauov pol na sve više energije, dok kod kvantne kromodinamike (QCD) vodi na hadronizaciju tvari na niskim energijama (jako vezanje, ali ne beskonačno)

Izvori

[1] C. N. Yang and R. L. Mills, *Conservation of Isotopic Spin and Isotopic Gauge Invariance*, Phys. Rev. **96**, 191 (1954).
[2] Michael E. Peskin, Dan V. Schroeder, *An Introduction to Quantum Field Theory*, (Addison-Wesley Pub. Co, 1995).
[3] H. F. Jones, *Groups, Representations and Physics*, (2nd ed., IOP Publishing, 1998).
[4] Faddeev, L. D.; Popov, V., *Feynman diagrams for the Yang-Mills field*, Phys. Lett. B. **25** (1): 29. (1967).
[5] J. Schwinger, *The Quantum Theory of Fields I*, Physical Review. 82 (6): 914–917 (1951).
[6] S. Weinberg, *The Quantum Theory of Fields*, (Cambridge University Press, 1995.)

Izvori

[7] Hooft, G. 't; Veltman, M. *Regularization and renormalization of gauge fields* Nuclear Physics B, **44** (1): 189–213 (1972.)
[8] Callan, Curtis G. *Broken Scale Invariance in Scalar Field Theory* Physical Review D. American Physical Society (APS). **2** (8): 1541–1547 (1970.)
[9] Symanzik, K. *Small distance behaviour in field theory and power counting* Communications in Mathematical Physics. Springer Science and Business Media LLC. **18** (3): 227–246 (1970.)
[10] G. F. Giudice; G. Isidori; A. Salvio; A. Strumia *Softened Gravity and the Extension of the Standard Model up to Infinite Energy* Journal of High Energy Physics. **2015** (2): 137. (2015)