f) $f: M_2(\mathbb{R}) \to \mathbb{R}$

$$f(X) = \operatorname{tr} X$$

$$f\left(\alpha \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \beta \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right)$$

$$= \alpha a_1 + \beta a_2 + \alpha d_1 + \beta d_2 =$$

$$= \alpha (a_1 + d_1) + \beta (a_2 + d_2) =$$

$$= \alpha f\left(\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}\right) + \beta f\left(\begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}\right)$$

$$\Rightarrow f \in (M_2(\mathbb{R}))^*$$

$$\Big\{ \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{E_1}, \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{E_2}, \underbrace{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}}_{E_3}, \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{E_4} \Big\} \text{ kanonska}$$

$$f = \alpha E_1^* + \beta E_2^* + \gamma E_3^* + \delta E_4^*$$

$$\left.\begin{aligned} f(E_1) = \alpha &\Longrightarrow \alpha = \operatorname{tr}(E_1) = 1 \\ f(E_2) = \beta &\Longrightarrow \beta = \operatorname{tr}(E_2) = 0 \\ f(E_3) = \gamma &\Longrightarrow \gamma = \operatorname{tr}(E_3) = 0 \\ f(E_4) = \delta &\Longrightarrow \delta = \operatorname{tr}(E_4) = 1 \end{aligned}\right\} \Rightarrow f = E_1^* + E_4^*$$