Alps - Adria Seminar

(7th meeting)

April 13, 2024

Department of Mathematics
Faculty od Science
University of Zagreb
Bijenička cesta 30
10000 Zagreb, Croatia

Schedule

All talks will be in Lecture Room A002

10:00–10:25 Vanja Wagner: *On the density of compactly supported smooth functions on $D \subset \mathbf{R}^d$ in Besov-type spaces of generalized smoothness $H^{\psi,1}(D)$*

10:30–10:55 Matej Brešar: *Zero product determined algebras*

11:00–11:25 Nick Rome: *Magic squares of squares and the Hardy–Littlewood method*

Coffee break

12:00–12:25 Matija Kazalicki: *Ranks of elliptic curves and neural networks*

12:30–12:55 Bojan Kuzma: *Spectrum preservers on unbounded operators*

13:00–13:25 Laura Cossu: *Classical Results and New Approaches to the Idempotent Factorization of Matrices*

14:00– Lunch at Tvornica pljeskavica Kosta, Savska 107/1

Abstracts

Zero product determined algebras

Matej Brešar

A not necessarily associative algebra A over a field F is said to be zero product determined if every bilinear functional $\varphi : A \times A \to F$ with the property that $xy = 0$ implies $\varphi(x, y) = 0$ is of the form $\varphi(x, y) = \tau(xy)$ for some linear functional τ on A. These algebras have been studied in pure algebra as well as in functional analysis where one additionally assumes that φ and τ are continuous.

The purpose of the talk is to give a brief introduction to the theory of zero product determined algebras, with an emphasis on the motivation, and at the end present a certain generalization, recently proposed in our joint work with Ž. Bajuk, P. Fagundes, and A. Ioppolo, in which the role of the product xy is replaced by a multilinear polynomial $f(x_1, \ldots, x_n)$.

Classical Results and New Approaches to the Idempotent Factorization of Matrices

Laura Cossu

A classical open problem in ring theory is to characterize integral domains R such that every singular matrix over R can be expressed as a product of idempotent matrices. The significance of this problem lies in its connections to other major open questions, including the classification of integral domains whose general linear groups are generated by elementary matrices and those satisfying weaker versions of the Euclidean algorithm. Notably, over a Bézout domain (i.e., a domain in which every finitely generated ideal is principal), a singular matrix can be decomposed into idempotent factors if and only if every invertible matrix can be decomposed into elementary matrices, and this holds true if and only if the domain admits a weak algorithm. In this seminar, we will provide an overview of classical results concerning the idempotent factorization of matrices, along with recent developments in the field. In particular, we will briefly introduce a new approach to matrix factorization grounded in factorization theory.

Ranks of elliptic curves and neural networks

Matija Kazalicki

Determining the algebraic rank of an elliptic curve E/Q is challenging, often relying on heuristics to estimate the analytic rank, which is conjecturally equal to the algebraic rank under the Birch and Swinnerton-Dyer conjecture. This talk presents a novel rank classification method utilizing deep convolutional neural networks (CNNs). The method takes the conductor of E and a sequence of Frobenius traces a_p as input to predict rank or detect "high" rank curves. Our method and eight simple neural network models utilizing Mestre-Nagao sums, commonly employed heuristics are compared. Results from evaluating both methods on the LMFDB dataset and a custom dataset show that CNNs outperform Mestre-Nagao sums on the LMFDB dataset while demonstrating comparable performance on the custom dataset. This is joint work with Domagoj Vlah.

Additionally, we will elaborate on an ongoing project with Zvonimir Bujanović and Lukas Novak. We'll explain how the recently observed phenomenon of murmurations of elliptic curves can help us understand fluctuations in the averages of Mestre-Nagao sums, leading to improved classification quality.

Spectrum preservers on unbounded operators

Bojan Kuzma

Unbounded, densely defined, closed operators are encountered in quantum physics when solving linear second-order partial differential equations with the method of variable separation (Sturm-Liouville problem) and as generators of strongly continuous one-parameter semigroups. Particularly important is their spectrum. We will present a recent result that classifies additive bijections preserving the spectrum of such operators and compare it to the classification of spectrum-preserving maps whose domain consists of all unbounded operators.

This is a joint work with G. Dolinar, J. Marovt, and E. Poon.

Magic squares of squares and the Hardy–Littlewood method

Nick Rome

I'll report on joint work in progress with Shuntaro Yamagishi establishing that there exist lots of magic squares all of whose entries are squares of integers. The proof uses the circle method developed by Hardy–Littlewood which allows one to count integer solutions to systems of equations by analysing exponential sums.

On the density of compactly supported smooth functions on $D \subset \mathbf{R}^d$ in Besov-type spaces of generalized smoothness $H^{\psi,1}(D)$

Vanja Wagner

Let $D \subset \mathbb{R}^d$ be an open d-set, i.e. such that there exist $c_1, c_2 > 0$ such that for every $x \in D$ and $r \in (0,1)$

$$c_1 r^d \leq \lambda(D \cap B(x,r)) \leq c_2 r^d.$$

Using the probabilistic potential-theoretical approach and Dirichlet form theory, we are revisiting the question of the density of compactly supported smooth functions $C_c^\infty(D)$ on D in Sobolev (or Besov) spaces $W^{s,2}(D)$ of fractional order $s \in (0,1)$, as well as their generalisations. These generalisations are connected to a nice class of Lévy characteristic exponents ψ via the generalised Slobodeckij seminorm

$$[f]_\psi = \left(\int_D \int_D (f(y) - f(x))^2 \frac{\psi(|x-y|^{-1})}{|x-y|^d} \, dy \, dx \right)^{\frac{1}{2}}.$$

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