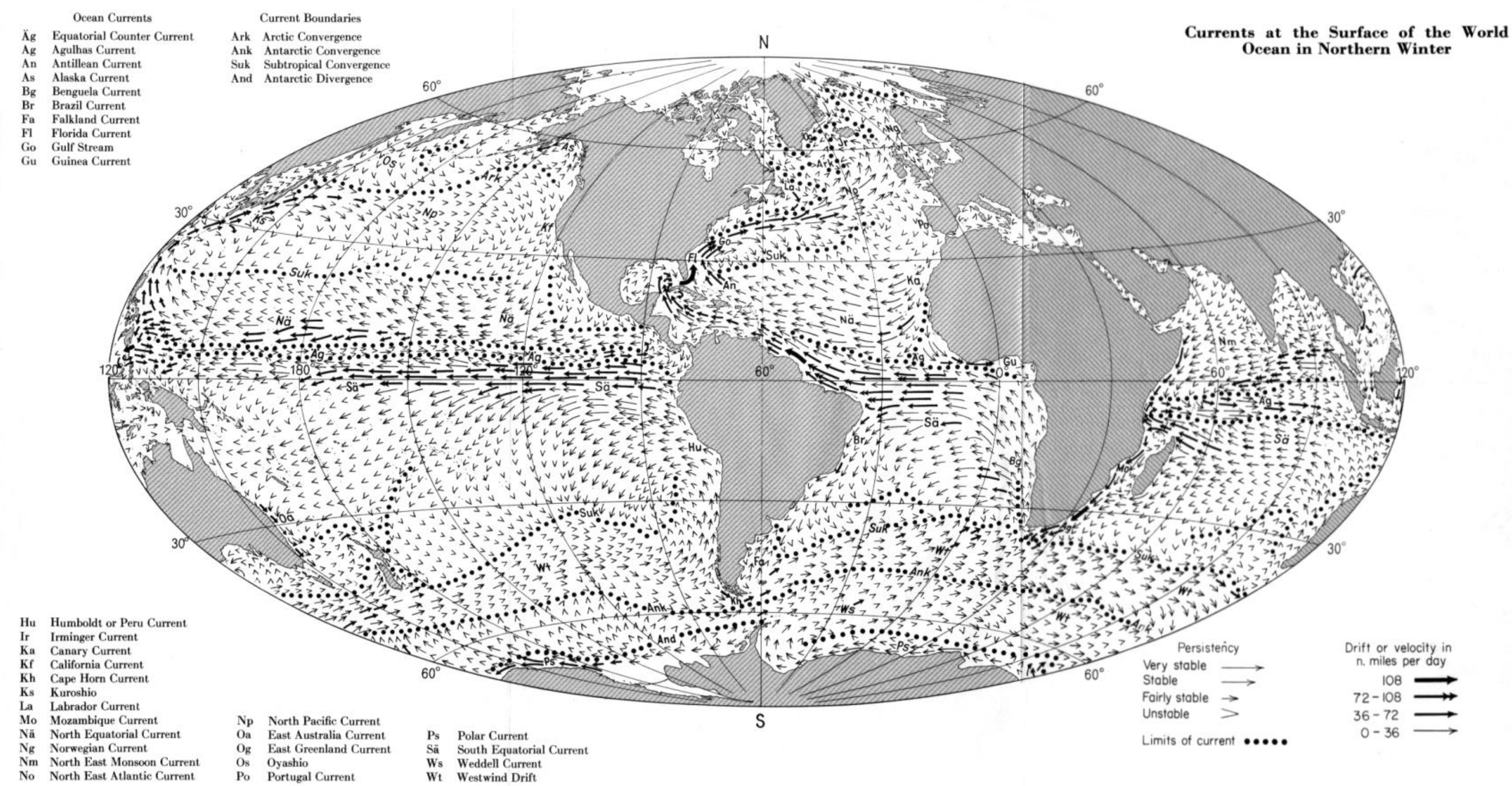
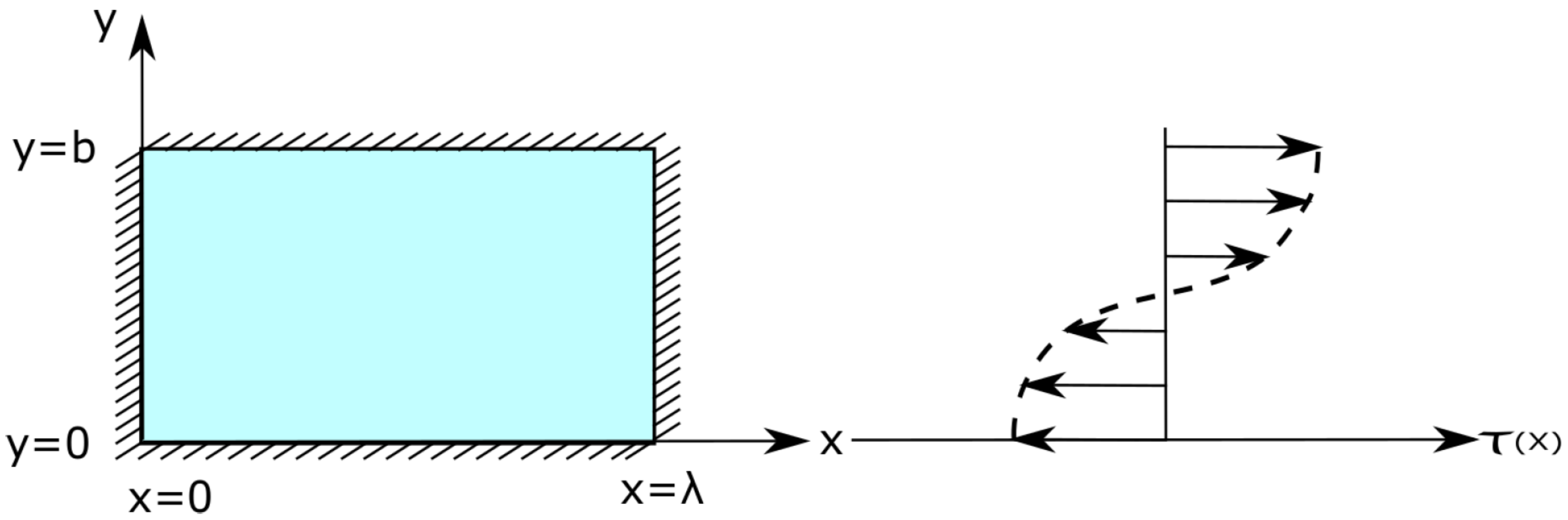
Drugo predavanje
(18. ožujka 2022.)

1.3. Stommelov model
(H. Stommel, 1948)



Dietrich, 1963.

Shematizacija problema



$$\tau_x = -F \cos \frac{\pi y}{b}, \qquad \tau_y = 0$$

Početne jednadžbe

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv + \frac{1}{\rho}\frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = -fu + \frac{1}{\rho}\frac{\partial}{\partial z}\left(A\frac{\partial v}{\partial z}\right)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Jednadžbe nakon integracije duž vertikale

$$g\left(\zeta + D\right)\frac{\partial \zeta}{\partial x} = f\left(\zeta + D\right)V + \frac{1}{\rho}\tau_x - RU$$

$$g\left(\zeta + D\right)\frac{\partial \zeta}{\partial y} = -f\left(\zeta + D\right)U - RV$$

$$\frac{\partial}{\partial x}\left[\left(\zeta + D\right)U\right] + \frac{\partial}{\partial y}\left[\left(\zeta + D\right)V\right] = 0$$

Poprečnim deriviranjem i oduzimanjem jednadžba gibanja te uvažavanjem jednadžbe kontinuiteta dobije se:

$$V\frac{D}{R}\frac{df}{dy} + \frac{1}{R\rho}\frac{\partial \tau_x}{\partial y} + \left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) = 0$$

Jednadžba kontinuiteta implicira da se može uvesti strujna funkcija:

$$U = \frac{\partial \Psi}{\partial y}, \quad V = -\frac{\partial \Psi}{\partial x}$$

Konačno se dobiva:

$$\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}\right)+\alpha\frac{\partial\Psi}{\partial x}=\gamma\sin\frac{\pi y}{b}$$

$$\alpha=\frac{D}{R}\frac{df}{dy},\qquad \gamma=\frac{F}{\rho R}\frac{\pi}{b}$$

što treba riješiti uz pripadne rubne uvjete:

$$\Psi\left(0,y\right)=\Psi\left(\lambda,y\right)=\Psi\left(x,0\right)=\Psi\left(x,b\right)=0$$

Određivanjem partikularnog rješenja te općeg rješenja homogenizirane jednadžbe dobije se:

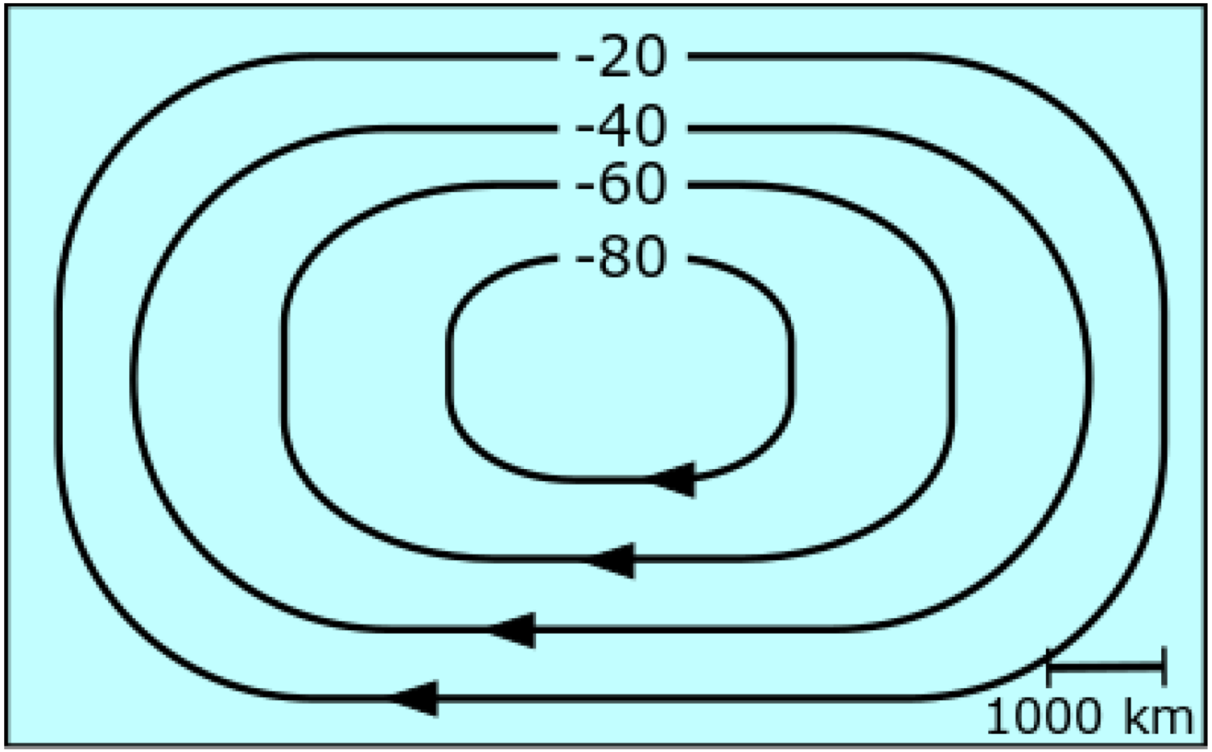
$$\Psi = \gamma \frac{b^2}{\pi^2} \sin \frac{\pi y}{b} \left[\frac{\left(1 - e^{k_2 \lambda}\right) e^{k_1 x} + \left(e^{k_1 \lambda} - 1\right) e^{k_2 x}}{e^{k_1 \lambda} - e^{k_2 \lambda}} - 1 \right]$$

gdje je:

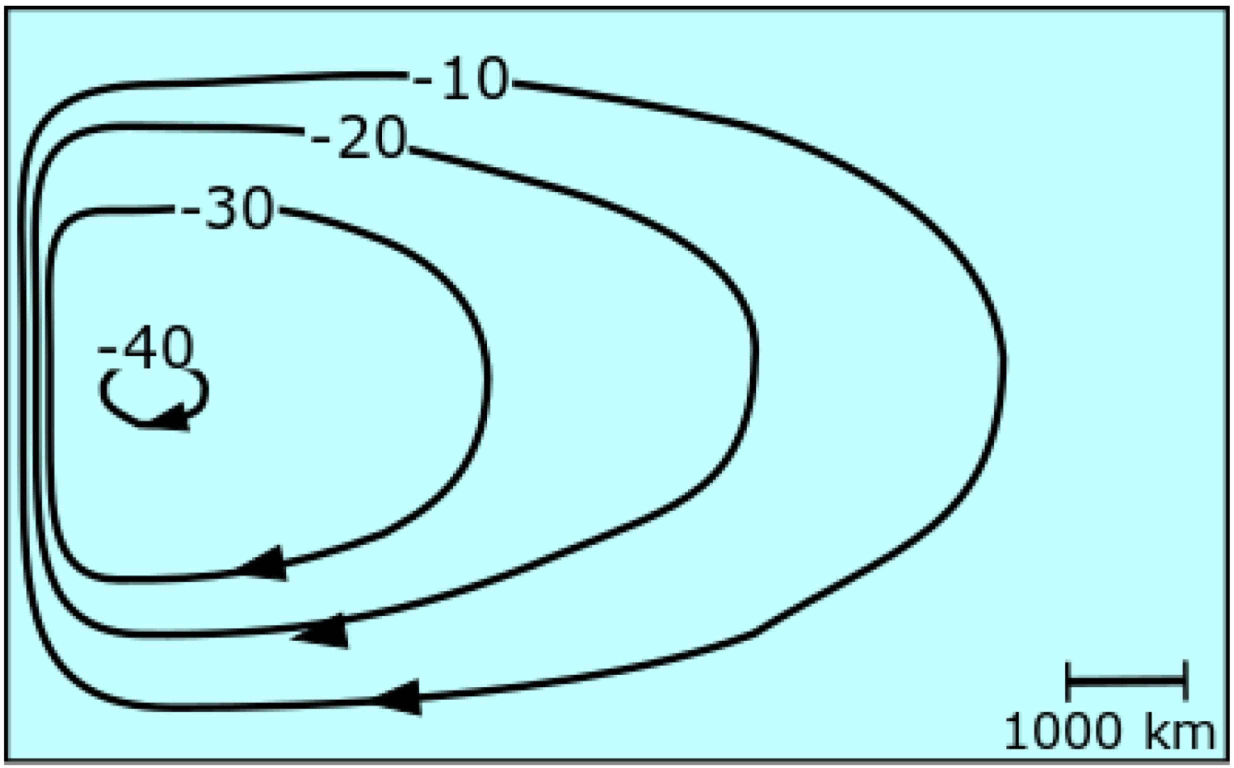
$$k_1 = -\frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \frac{\pi^2}{b^2}}$$

$$k_2 = -\frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} + \frac{\pi^2}{b^2}}$$

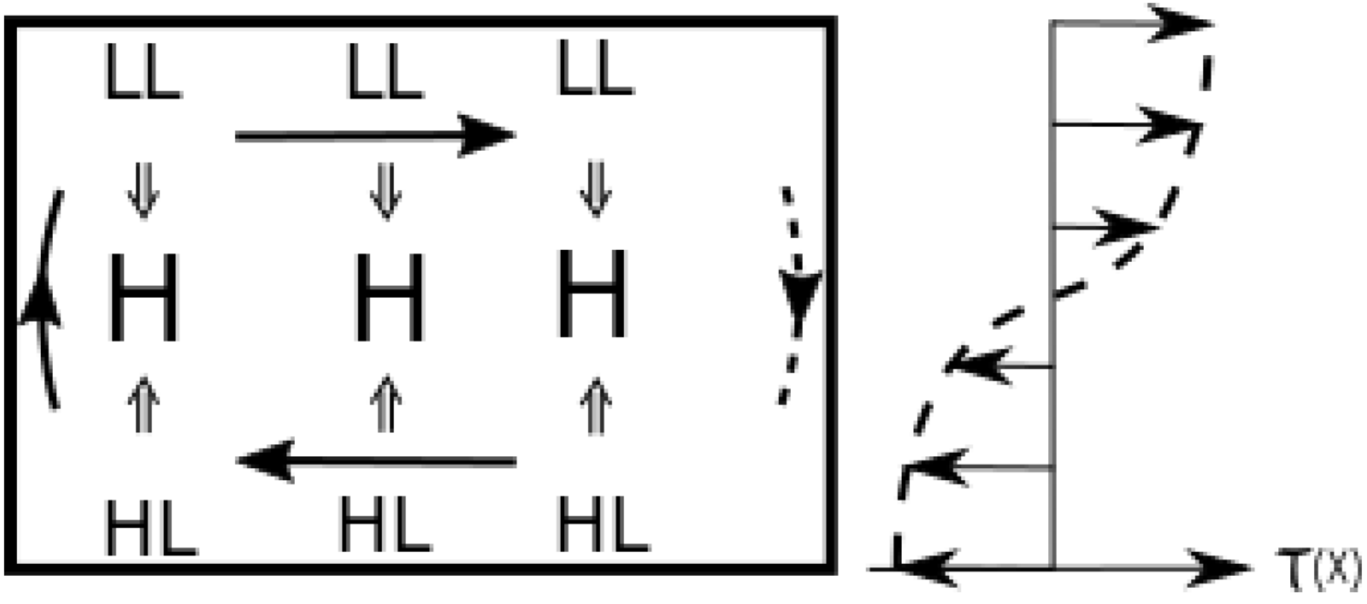
Slučaj nerotirajućeg oceana ($f \to 0$) i uniformno rotirajućeg oceana ($df/dy \to 0$)



Slučaj oceana u β-ravnini



Interpretacija



Veronis, 1981.

Usporedba Stommelovog modela:

$$\left(\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2}\right) + \alpha\frac{\partial\Psi}{\partial x} = \gamma\sin\frac{\pi y}{b}$$

$$\alpha = \frac{D}{R}\frac{df}{dy}, \quad \gamma = \frac{F}{\rho R}\frac{\pi}{b}$$

i Sverdrupovog modela:

$$\frac{df}{dy}DV + \frac{1}{\rho}\frac{\partial\tau_x}{\partial y} = 0$$