Peto predavanje
(8. travnja 2022.)

2.3. Model ruske škole (V. B. Štokman, 1941, 1953, i dr.)

Početne jednadžbe

$$\frac{\partial p}{\partial x} = A\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial p}{\partial y} = A\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial p}{\partial z} = -g\rho$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,,$$

Jednadžbe nakon integracije duž vertikale (1)

$$p = p_a + g\rho(\zeta - z)$$

$$g\rho \frac{\partial \zeta}{\partial x} = A \frac{\partial^2 u}{\partial z^2}$$

$$g\rho \frac{\partial \zeta}{\partial y} = A \frac{\partial^2 v}{\partial z^2}.$$

$$-g\rho \frac{\partial \zeta}{\partial x} z = \tau_x - A \frac{\partial u}{\partial z}$$

$$-g\rho \frac{\partial \zeta}{\partial y} z = \tau_y - A \frac{\partial v}{\partial z}$$

$$u = \frac{\tau_x}{A}(z + D) + \frac{g\rho}{A} \frac{\partial \zeta}{\partial x} \frac{z^2 - D^2}{2}$$

$$v = \frac{\tau_y}{A}(z + D) + \frac{g\rho}{A} \frac{\partial \zeta}{\partial y} \frac{z^2 - D^2}{2}$$

Jednadžbe nakon integracije duž vertikale (2)

$$\mathbf{U} = \frac{\tau_x D^2}{2A} - \frac{g\rho D^3}{3A}\frac{\partial \zeta}{\partial x}$$

$$\mathbf{V} = \frac{\tau_y D^2}{2A} - \frac{g\rho D^3}{3A}\frac{\partial \zeta}{\partial y}$$

$$\frac{\partial \mathbf{U}}{\partial x} + \frac{\partial \mathbf{V}}{\partial y} = 0$$

$$\mathbf{U} = -\frac{\partial \Psi}{\partial y}, \qquad \mathbf{V} = \frac{\partial \Psi}{\partial x},$$

$$\frac{\partial \zeta}{\partial x} = \frac{3\tau_x}{2g\rho D} + \frac{3A}{g\rho D^3}\frac{\partial \Psi}{\partial y}$$

$$\frac{\partial \zeta}{\partial y} = \frac{3\tau_y}{2g\rho D} - \frac{3A}{g\rho D^3}\frac{\partial \Psi}{\partial x}$$

Konačni izraz (uz $\Psi = 0$ duž obala):

$$\frac{\partial}{\partial x}\left(\frac{1}{D^3}\frac{\partial\Psi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{D^3}\frac{\partial\Psi}{\partial y}\right)=\frac{1}{2A}\left[\frac{\partial}{\partial x}\left(\frac{\tau_y}{D}\right)-\frac{\partial}{\partial y}\left(\frac{\tau_x}{D}\right)\right]$$

Redoslijed rješavanja:

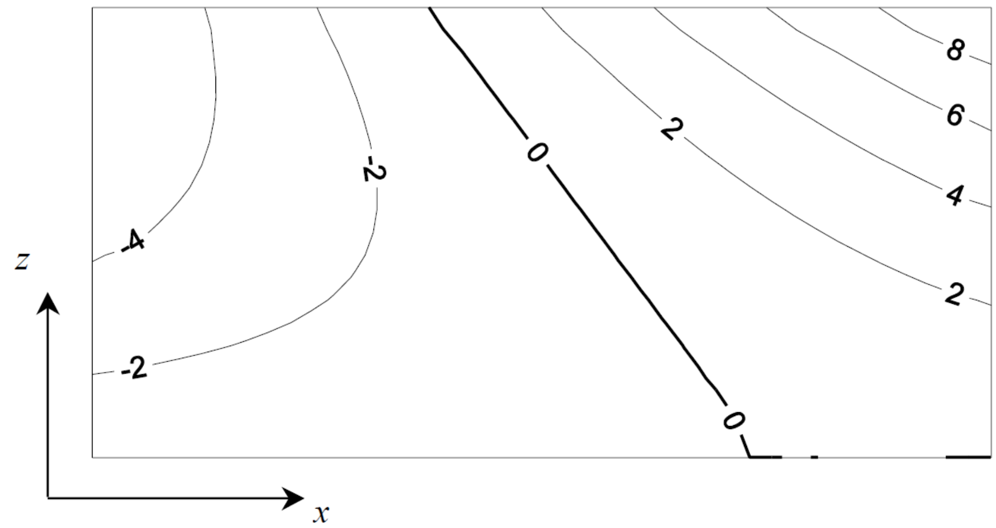
$$\psi$$

$$\frac{\partial\zeta}{\partial x},\frac{\partial\zeta}{\partial y}$$

$$u, v$$

Prvi slučaj: ravno dno (*vjetar promjenjiv*)

$$\frac{d^2\Psi}{dx^2} = \frac{D^2}{2A}\frac{d\tau_y}{dx}$$

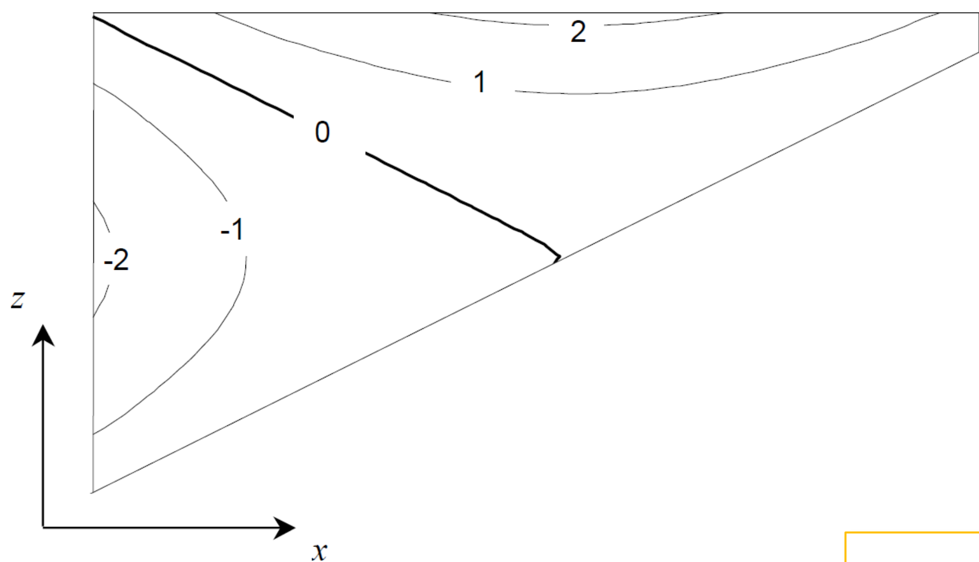


$$\mathbf{V} = \frac{d\Psi}{dx} = \frac{D^2}{2A}\left(\tau_y - \bar{\tau}_y\right)$$

$$v = \frac{\tau_y}{A}\left(z + D\right) + \frac{3}{4}\frac{\bar{\tau}_y}{AD}\left(z^2 - D^2\right)$$

Drugi slučaj: uniforman vjetar (*dno nagnuto*)

$$\frac{d}{dx}\left(\frac{1}{D^3}\frac{d\Psi}{dx}\right)=\frac{\tau_y}{2A}\frac{d}{dx}\left(\frac{1}{D}\right)$$



$$\mathbf{V}=\frac{d\Psi}{dx}=\frac{\tau_y}{2A}\left(D^2-\frac{\overline{D^2}}{\overline{D^3}}D^3\right)$$

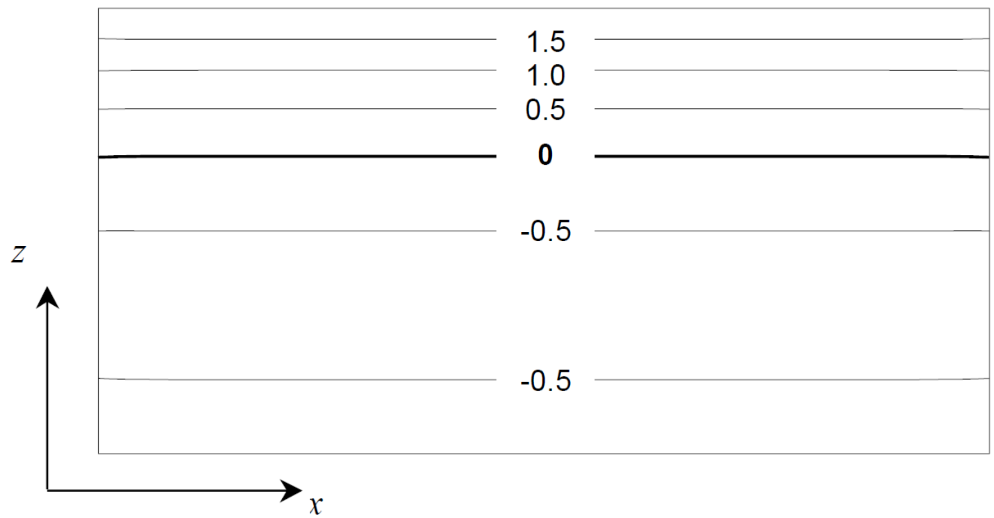
$$v=\frac{\tau_y}{A}\left(z+D\right)+\frac{3}{4}\frac{\tau_y}{A}\frac{\overline{D^2}}{\overline{D^3}}\left(z^2-D^2\right)$$

Treći slučaj: ravno dno i uniforman vjetar

$$\frac{d^2\Psi}{dx^2} = 0$$

$$\mathbf{V} = \frac{d\Psi}{dx} = C$$

$$C = 0$$



$$v = \frac{\tau_y}{4A}\left(D + 4z + 3\frac{z^2}{D}\right)$$