BOLTZMANNOVA RASPODJELA

DODATNI ZADACI - RJEŠENJA

10. Izračunajte relativnu populacijsku razliku ($\Delta N/N_o$) za protone u polju od a) 0,30 T b) 1,5 T i c) 10 T pri 25 °C (γ = 2,6752·10^8 rad T^{-1} s^{-1}).

$$\vartheta = 25\,^{\circ}\mathrm{C} \quad \Rightarrow \quad T = 298{,}15\ \mathrm{K} \qquad \gamma(^1\mathrm{H}) = 2{,}6752 \cdot 10^8\ \mathrm{rad\ T^{-1}\ s^{-1}}$$

a) $B_o = 0{,}30\ \mathrm{T}$

$$\nu_L = \frac{\gamma B_o}{2\pi} = \frac{2{,}6752 \cdot 10^8\ \mathrm{rad\ T^{-1}\ s^{-1}} \cdot 0{,}30\ \mathrm{T}}{2\pi} = 12{,}77 \cdot 10^6\ \mathrm{Hz}$$

$$\Delta E = h\,\nu_L = 6{,}626 \cdot 10^{-34}\ \mathrm{J\,s} \cdot 12{,}77 \cdot 10^6\ \mathrm{Hz} = 8{,}463 \cdot 10^{-27}\ \mathrm{J}$$

$$\frac{\Delta N}{N_o} = 1 - e^{-\frac{\Delta E}{k\,T}} = 2{,}057 \cdot 10^{-6}$$

b) $B_o = 1{,}5\ \mathrm{T}$

$$\nu_L = \frac{\gamma B_o}{2\pi} = \frac{2{,}6752 \cdot 10^8\ \mathrm{rad\ T^{-1}\ s^{-1}} \cdot 1{,}5\ \mathrm{T}}{2\pi} = 63{,}87 \cdot 10^6\ \mathrm{Hz}$$

$$\Delta E = h\,\nu_L = 6{,}626 \cdot 10^{-34}\ \mathrm{J\ s} \cdot 63{,}87 \cdot 10^6\ \mathrm{Hz} = 4{,}232 \cdot 10^{-26}\ \mathrm{J}$$

$$\frac{\Delta N}{N_o} = 1 - e^{-\frac{\Delta E}{k\,T}} = 1{,}028 \cdot 10^{-5}$$

c) $B_o = 10\ \mathrm{T}$

$$\nu_L = \frac{\gamma B_o}{2\pi} = \frac{2{,}6752 \cdot 10^8\ \mathrm{rad\ T^{-1}\ s^{-1}} \cdot 10\ \mathrm{T}}{2\pi} = 425{,}77 \cdot 10^6\ \mathrm{Hz}$$

$$\Delta E = h\,\nu_L = 6{,}626 \cdot 10^{-34}\ \mathrm{J\ s} \cdot 425{,}77 \cdot 10^6\ \mathrm{Hz} = 2{,}821 \cdot 10^{-25}\ \mathrm{J}$$

$$\frac{\Delta N}{N_o} = 1 - e^{-\frac{\Delta E}{k\,T}} = 6{,}856 \cdot 10^{-5}$$

11. Izračunajte relativnu populacijsku razliku ($\Delta N/N_o$) za jezgre ^{13}C u polju od a) 0,30 T b) 1,5 T i c) 10 T pri 25 °C? ($\gamma = 6{,}7283 \cdot 10^7$ rad T^{-1} s^{-1}).

$$\vartheta = 25\,^{\circ}\mathrm{C} \quad \Rightarrow \quad T = 298{,}15\,\mathrm{K} \qquad \gamma(^{13}\mathrm{C}) = 6{,}7283 \cdot 10^{7}\ \mathrm{rad\ T^{-1}\ s^{-1}}$$

a) $B_o = 0{,}30\,\mathrm{T}$

$$\nu_L = \frac{\gamma B_o}{2\pi} = \frac{6{,}7283 \cdot 10^{7}\ \mathrm{rad\ T^{-1}\ s^{-1}} \cdot 0{,}30\,\mathrm{T}}{2\pi} = 3{,}21 \cdot 10^{6}\ \mathrm{Hz}$$

$$\Delta E = h\,\nu_L = 6{,}626 \cdot 10^{-34}\ \mathrm{J\,s} \cdot 3{,}21 \cdot 10^{6}\ \mathrm{Hz} = 2{,}129 \cdot 10^{-27}\ \mathrm{J}$$

$$\frac{\Delta N}{N_o} = 1 - e^{-\frac{\Delta E}{k\,T}} = 5{,}173 \cdot 10^{-7}$$

b) $B_o = 1{,}5\ \mathrm{T}$

$$\nu_L = \frac{\gamma B_o}{2\pi} = \frac{6{,}7283 \cdot 10^7\ \mathrm{rad\ T^{-1}\ s^{-1}} \cdot 1{,}5\ \mathrm{T}}{2\pi} = 16{,}06 \cdot 10^6\ \mathrm{Hz}$$

$$\Delta E = h\,\nu_L = 6{,}626 \cdot 10^{-34}\ \mathrm{J\ s} \cdot 16{,}06 \cdot 10^6\ \mathrm{Hz} = 1{,}064 \cdot 10^{-26}\ \mathrm{J}$$

$$\frac{\Delta N}{N_o} = 1 - e^{-\frac{\Delta E}{k\,T}} = 2{,}587 \cdot 10^{-6}$$

c) $B_o = 10\ \mathrm{T}$

$$\nu_L = \frac{\gamma B_o}{2\pi} = \frac{6{,}7283 \cdot 10^7\ \mathrm{rad\ T^{-1}\ s^{-1}} \cdot 10\ \mathrm{T}}{2\pi} = 107{,}08 \cdot 10^6\ \mathrm{Hz}$$

$$\Delta E = h\,\nu_L = 6{,}626 \cdot 10^{-34}\ \mathrm{J\ s} \cdot 107{,}08 \cdot 10^6\ \mathrm{Hz} = 7{,}095 \cdot 10^{-26}\ \mathrm{J}$$

$$\frac{\Delta N}{N_o} = 1 - e^{-\frac{\Delta E}{k\,T}} = 1{,}724 \cdot 10^{-5}$$

12. Izračunajte omjer broja molekula Cl_2 u osnovnom i prvom pobuđenom vibracijskom stanju pri a) 298 K i b) 500 K. Valni broj vibracije molekule Cl_2 iznosi 559,7 cm^{-1}.

$$\tilde{\nu}(\mathrm{Cl_2}) = 559{,}7\ \mathrm{cm^{-1}} = 559{,}7 \cdot 10^2\ \mathrm{m^{-1}}$$

$$\Delta E = h\,c\,\tilde{\nu} = 6{,}626 \cdot 10^{-34}\ \mathrm{J\,s} \cdot 3 \cdot 10^8\ \mathrm{m\,s^{-1}} \cdot 559{,}7 \cdot 10^2\ \mathrm{m^{-1}} = 1{,}113 \cdot 10^{-20}\ \mathrm{J}$$

a) $T = 298\ \mathrm{K}$

$$\frac{N_\mathrm{j}}{N_\mathrm{o}} = e^{-\frac{\Delta E}{k\,T}} = 0{,}067$$

b) $T = 500\ \mathrm{K}$

$$\frac{N_\mathrm{j}}{N_\mathrm{o}} = e^{-\frac{\Delta E}{k\,T}} = 0{,}199$$

13. Izračunajte omjer broja molekula Br_2 u osnovnom i prvom pobuđenom vibracijskom stanju pri a) 298 K i b) 800 K. Valni broj vibracije molekule Br_2 iznosi 321,0 cm^{-1}.

$$\widetilde{\nu}(\mathrm{Br_2}) = 321{,}0\ \mathrm{cm^{-1}} = 321{,}0 \cdot 10^2\ \mathrm{m^{-1}}$$

$$\Delta E = h\,c\,\widetilde{\nu} = 6{,}626 \cdot 10^{-34}\ \mathrm{J\,s} \cdot 3 \cdot 10^8\ \mathrm{m\,s^{-1}} \cdot 321{,}0 \cdot 10^2\ \mathrm{m^{-1}} = 6{,}381 \cdot 10^{-21}\ \mathrm{J}$$

a) $T = 298\ \mathrm{K}$

$$\frac{N_\mathrm{j}}{N_\mathrm{o}} = e^{-\frac{\Delta E}{k\,T}} = 0{,}212$$

b) $T = 800\ \mathrm{K}$

$$\frac{N_\mathrm{j}}{N_\mathrm{o}} = e^{-\frac{\Delta E}{k\,T}} = 0{,}561$$

14. Za Na^+ (589,3 nm) i Mg^{2+} (457,1 nm) ione usporedite omjer broja iona u 3p pobuđenom stanju u odnosu na osnovno pri a) 2100 K; b) 2900 K i c) u induktivno spregnutoj plazmi pri 6000 K.

$\tilde{\nu}(\mathrm{Na}^+) = 589{,}3\ \mathrm{nm} = 589{,}3 \cdot 10^{-9}\ \mathrm{m}$ 3s $\Rightarrow$ $g_o = 2$

$\tilde{\nu}(\mathrm{Mg}^{2+}) = 457{,}1\ \mathrm{nm} = 457{,}1 \cdot 10^{-9}\ \mathrm{m}$ 3p $\Rightarrow$ $g_j = 6$

$$\Delta E(\mathrm{Na}^+) = \frac{h\,c}{\lambda(\mathrm{Na}^+)} = \frac{6{,}626 \cdot 10^{-34}\ \mathrm{J\,s} \cdot 3 \cdot 10^{8}\ \mathrm{m\,s^{-1}}}{589{,}3 \cdot 10^{-9}\ \mathrm{m}} = 3{,}373 \cdot 10^{-19}\ \mathrm{J}$$

$$\Delta E(\mathrm{Mg}^{2+}) = \frac{h\,c}{\lambda(\mathrm{Mg}^{2+})} = \frac{6{,}626 \cdot 10^{-34}\ \mathrm{J\,s} \cdot 3 \cdot 10^{8}\ \mathrm{m\,s^{-1}}}{457{,}1 \cdot 10^{-9}\ \mathrm{m}} = 4{,}349 \cdot 10^{-19}\ \mathrm{J}$$

a) $T = 2100\ \mathrm{K}$

Na^+ $\dfrac{N_j}{N_o} = \dfrac{g_j}{g_o} e^{-\frac{\Delta E}{k\,T}} = 2{,}643 \cdot 10^{-5}$ **Mg^{2+}** $\dfrac{N_j}{N_o} = \dfrac{g_j}{g_o} e^{-\frac{\Delta E}{k\,T}} = 9{,}123 \cdot 10^{-7}$

b) $T = 2900\,\mathrm{K}$

Na⁺ $\dfrac{N_\mathrm{j}}{N_\mathrm{o}} = \dfrac{g_\mathrm{j}}{g_\mathrm{o}} e^{-\frac{\Delta E}{k\,T}} = 6{,}555 \cdot 10^{-4}$ **Mg²⁺** $\dfrac{N_\mathrm{j}}{N_\mathrm{o}} = \dfrac{g_\mathrm{j}}{g_\mathrm{o}} e^{-\frac{\Delta E}{k\,T}} = 5{,}727 \cdot 10^{-5}$

c) $T = 6000\,\mathrm{K}$

Na⁺ $\dfrac{N_\mathrm{j}}{N_\mathrm{o}} = \dfrac{g_\mathrm{j}}{g_\mathrm{o}} e^{-\frac{\Delta E}{k\,T}} = 0{,}051$ **Mg²⁺** $\dfrac{N_\mathrm{j}}{N_\mathrm{o}} = \dfrac{g_\mathrm{j}}{g_\mathrm{o}} e^{-\frac{\Delta E}{k\,T}} = 0{,}016$