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MATEMATIČKA ANALIZA 2

Drugi kolokvij – 24. lipnja 2019.

- Dozvoljeno je koristiti samo pribor za pisanje i brisanje, te službene formule koje će student dobiti zajedno s kolokvijem.
- Rješenja će biti objavljena danas na web-stranici kolegija.
- Rezultati će biti objavljeni do nedjelje, 30. lipnja 2019. u 19 sati na web-stranici kolegija.
- Uvid u kolokvij održat će se u ponedjeljak, 1. srpnja 2019. u 13 sati u prostoriji 109.

Zadatak 1.

(a) (3 boda) Izračunajte integral

$$\int \sqrt{x^2 - 2x + 5}\, dx.$$

(b) (4 boda) Odredite sve $\alpha \in \mathbb{R}$ takve da nepravi integral

$$\int_1^{+\infty} \frac{x^\alpha}{|\sin x|^{1/3} + x}\, dx$$

konvergira.

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Zadatak 2. (6 bodova) Izračunajte volumen tijela nastalog rotacijom oko y osi lika omeđenog krivuljama $y = \frac{1}{1+x^2}$ i $y = \frac{x^2}{2}$.

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Zadatak 3.

(a) (4 boda) Ispitajte konvergenciju reda

$$\sum_{n=1}^{\infty} \frac{\ln(n^4 + 10)}{\sqrt{n^3 - \ln n}}.$$

(b) (2 boda) Neka je (a_n) niz pozitivnih realnih brojeva takav da red $\sum_{n=1}^{\infty} a_n$ konvergira. Dokažite da tada konvergira i red

$$\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}}.$$

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Zadatak 4.

(a) (3 boda) Razvijte u Maclaurinov red funkciju $f(x) = \frac{1}{2}\ln\frac{1+x}{1-x}$ i odredite radijus konvergencije dobivenog reda.

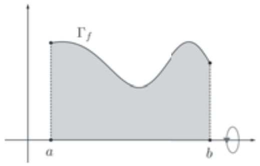
(b) (3 boda) Izračunajte sumu reda

$$\sum_{n=0}^{\infty}(-1)^n\frac{1+(2n)!}{3^{2n+1}(2n+1)!}.$$

VOLUMENI ROTACIJSKIH TIJELA

(1) Rotira oko x-osi

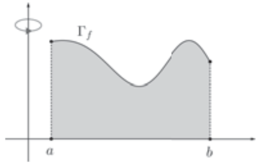
$f : [a, b] \to \mathbb{R}$



$$V_x = \pi \int_a^b f(x)^2 \, dx$$

(2) Rotira oko y-osi

$0 \le a < b, \quad f : [a, b] \to [0, +\infty\rangle$



$$V_y = 2\pi \int_a^b x \, f(x) \, dx$$

Taylorovi redovi

1. $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots \; = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}, \; x \in \mathbb{R}$

2. $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \ldots \; = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \; x \in \mathbb{R}$

3. $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!} + \ldots \; = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \; x \in \mathbb{R}$

4. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \displaystyle\sum_{n=0}^{\infty} x^n, \; |x| < 1$

5. $(1+x)^\alpha = 1 + \dbinom{\alpha}{1} x + \dbinom{\alpha}{2} x^2 + \ldots = \displaystyle\sum_{n=0}^{\infty} \binom{\alpha}{n} x^n, \; |x| < 1,$

   pri čemu je $\dbinom{\alpha}{n} = \dfrac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}, \; \dbinom{\alpha}{0} = 1$

6. $\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n-3)!!}{2^n \, n!} x^n, \; |x| < 1,$

   jer je $\dbinom{\frac{1}{2}}{n} = \dfrac{(-1)^{n-1}(2n-3)!!}{2^n \, n!}$

7. $\dfrac{1}{\sqrt{1+x}} = (1+x)^{-\frac{1}{2}} = 1 + \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2n-1)!!}{2^n n!} x^n, \; |x| < 1,$

   jer je $\dbinom{-\frac{1}{2}}{n} = \dfrac{(-1)^n(2n-1)!!}{2^n n!}$

8. Ako je P polinom stupnja m, onda je

   $P(x) = P(0) + \dfrac{P'(0)}{1!} x + \dfrac{P''(0)}{2!} x^2 + \ldots = \displaystyle\sum_{n=0}^{m} \frac{P^{(n)}(0)}{n!} x^n, \; |x| < 1$

9. $\ln(1+x) = \dfrac{x}{1} - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots = \displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}, \; |x| < 1$

10. $\operatorname{arctg} x = \dfrac{x}{1} - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \ldots = \displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}, \; |x| < 1$

11. $\arcsin x = \displaystyle\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n \, n!} \cdot \frac{x^{2n+1}}{2n+1}, \; |x| < 1$

Tablica integrala

$$\int dx = x + C$$

$$\int x^a \, dx = \frac{x^{a+1}}{a+1} + C \qquad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \qquad (a > 0, \; a \neq 1)$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \operatorname{ch} x \, dx = \operatorname{sh} x + C$$

$$\int \operatorname{sh} x \, dx = \operatorname{ch} x + C$$

$$\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C$$

$$\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \operatorname{Arsh} x + C = \ln\left(x + \sqrt{1+x^2}\right) + C$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \operatorname{Arch} x + C = \ln\left|x + \sqrt{x^2-1}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C \qquad (a > 0)$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \qquad (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right) + C \qquad (a > 0)$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C \qquad (a > 0)$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + C \qquad (a > 0)$$