Lokalna konvertibilnost u 1D magnetskim sustavima

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Pregled

- prepletenost u mnogočestičnim sustavima i definicija lokalne konvertibilnosti
- spin-½ XY lanac
- rezultati
  - ne-frustrirani lanac
  - frustrirani lanac
  - W stanja

Prepletenost u mnogočestičnim sustavima

- egzotični fenomeni u međudjelujućim mnogočestičnim kvantnim sustavima
- kvantna simulacija → veza s kvantnom teorijom informacija i kvantnim računalima
- Je li moguće efikasno klasično simulirati kvantni mnogočestični sustav?

→ proučavanje lokalne konvertibilnosti

Lokalna konvertibilnost

- složeni sustav AB – podjela na podsustave A, B
- lokalne operacije i klasična komunikacija (LOCC)
- Može li se transformacija $|\psi_1\rangle \rightarrow |\psi_2\rangle$ stanja sustava AB izvesti pomoću LOCC na podsustavima A i B?
- nemogućnost implicira dugodosežnu, neklasičnu koherenciju između A i B

Renyijeve entropije

- mjera prepletenosti među podsustavima

- Definicija: $S_\alpha = \frac{1}{1-\alpha} \log\left(\mathrm{Tr}(\rho_A^\alpha)\right)$ uz $\rho_A = \mathrm{Tr}_B \left|\psi\right\rangle \left\langle\psi\right|$

- Parametar $\alpha \in [0, \infty]$

$$S_0 = \log R$$

$$S_1 = -\mathrm{Tr}(\rho_A \log \rho_A)$$

$$S_\infty = -\log(\lambda_{max})$$

- Kriterij za lokalnu konvertibilnost: $S_\alpha(\rho_1) \geq S_\alpha(\rho_2)$ za sve Renyijeve entropije pri transformaciji $|\psi_1\rangle \to |\psi_2\rangle$

XY model

$$H = J\sum_{j=1}^{N}(\frac{1+\gamma}{2}\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{2}\sigma_j^y\sigma_{j+1}^y - h\sigma_j^z)$$

- egzaktno rješiv u jednoj dimenziji
- Jordan-Wignerova transformacija – mapiranje na fermione $\psi_j = \left(\prod_{l=1}^{j-1}\sigma_l^z\right)\sigma_j^+$
- Fourierov transformat $\psi_q = \frac{e^{-i\pi/4}}{\sqrt{N}}\sum_{j=1}^{N}e^{-iqj}\psi_j$
- Bogoljubovljeva rotacija $b_q = \cos\theta_q\psi_q + \sin\theta_q\psi_{-q}^\dagger \qquad \theta_q = \frac{1}{2}\arctan\left(\frac{\gamma\sin q}{h+\cos q}\right)$

Dijagonalizacija XY lanca

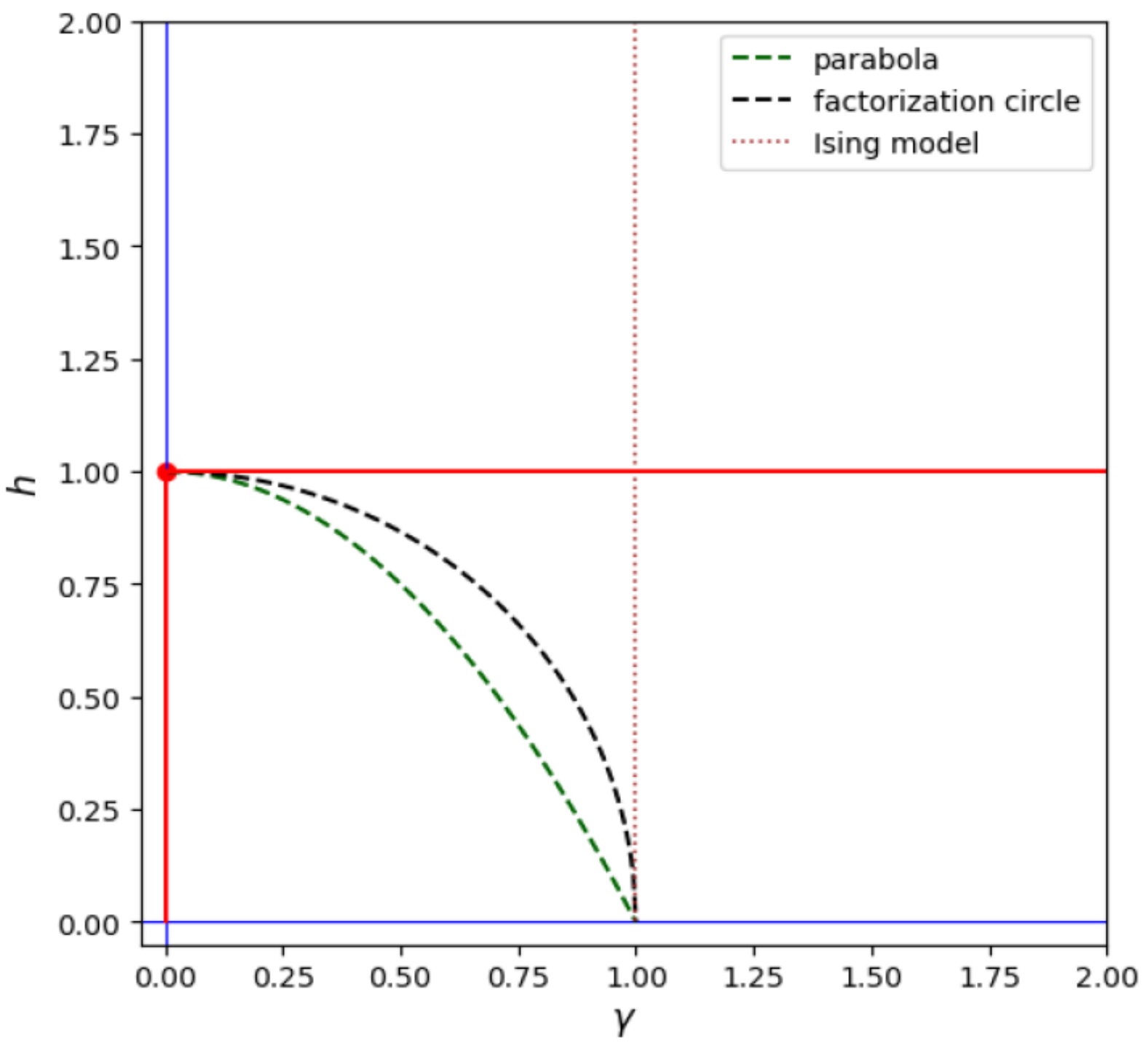
- dijagonalizirani Hamiltonijan $H = \frac{1+\Pi_z}{2} H^+ \frac{1+\Pi_z}{2} + \frac{1-\Pi_z}{2} H^- \frac{1-\Pi_z}{2}$

$$H^- = \sum_{q \in \Gamma^- - \{0\}} \Lambda(q)(b_q^\dagger b_q - \frac{1}{2}) + \epsilon(0)(b_0^\dagger b_0 - \frac{1}{2}) \qquad H^+ = \sum_{q \in \Gamma^+ - \{\pi\}} \Lambda(q)(b_q^\dagger b_q - \frac{1}{2}) + \epsilon(\pi)(b_\pi^\dagger b_\pi - \frac{1}{2})$$

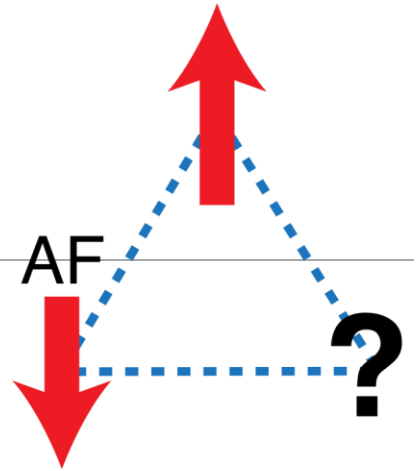
- Paritet $\Pi_z = \prod_{i=1}^{N} \sigma_i^z$
- disperzijska relacija $\Lambda(q) = |h + \cos q + i\gamma \sin q|; \quad q \neq 0, \pi$

$$\epsilon(0) = h + 1, \quad \epsilon(\pi) = h - 1$$

Fazni dijagram



Frustracija



- nemoguće istrovremeno minimizirati sve članove u Hamiltonijanu
- veća degeneracija osnovnog stanja
- kvantni fazni prijelaz
- Spinski lanac: antiferomagnetske interakcije (J>0), periodični rubni uvjeti $\sigma_j^\alpha = \sigma_{j+N}^\alpha$, neparan broj spinova N

- Kako utječe na lokalnu konvertibilnost?

Korelacijska matrica

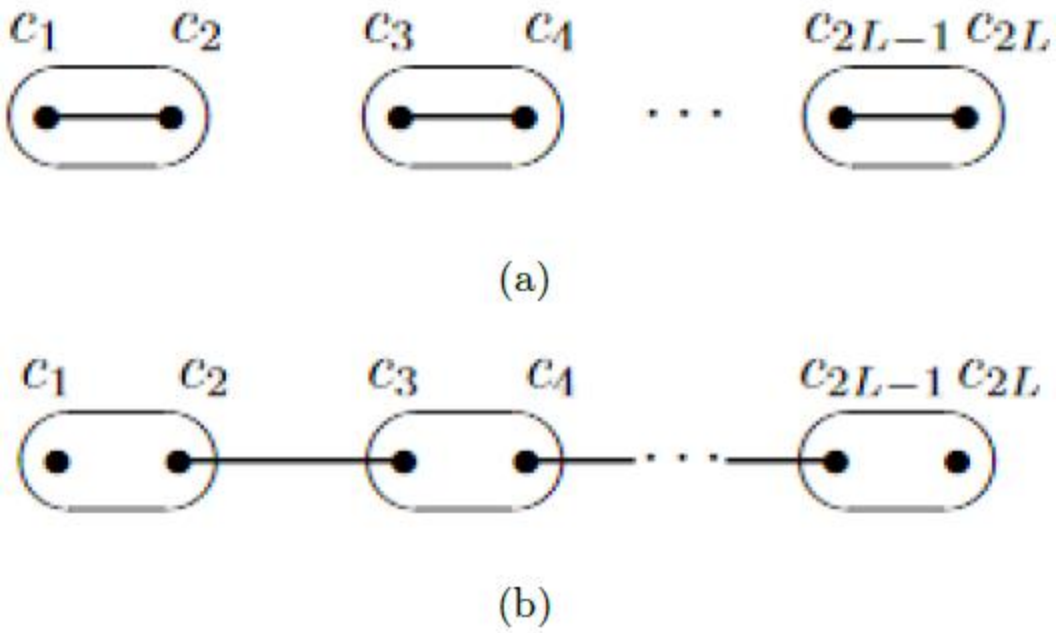
- Izvrjednjavanje Renyijevih entropija moguće iz korelacijske matrice s $O(L^2)$ elemenata
- Majorana fermioni $A_i = \psi_i^\dagger + \psi_i, \quad B_i = i(\psi_i - \psi_i^\dagger)$
- Elementi matrice

$$C_{2i,2j} = \langle A_i A_j \rangle - \delta_{i,j} \quad C_{2i,2j+1} = \langle A_i B_j \rangle$$
$$C_{2i+1,2j} = \langle B_i A_j \rangle \quad C_{2i+1,2j+1} = \langle B_i B_j \rangle - \delta_{i,j}$$

- Renyijeve entropije $S_\alpha = \frac{1}{1-\alpha} \sum_{j=1}^{L} \log\left((\frac{1+\nu_j}{2})^\alpha + (\frac{1-\nu_j}{2})^\alpha \right)$
- Svojstvene vrijednosti matrice gustoće $\{\lambda_l\} = \prod_{j=1}^{L} (\frac{1 \pm \nu_j}{2})$

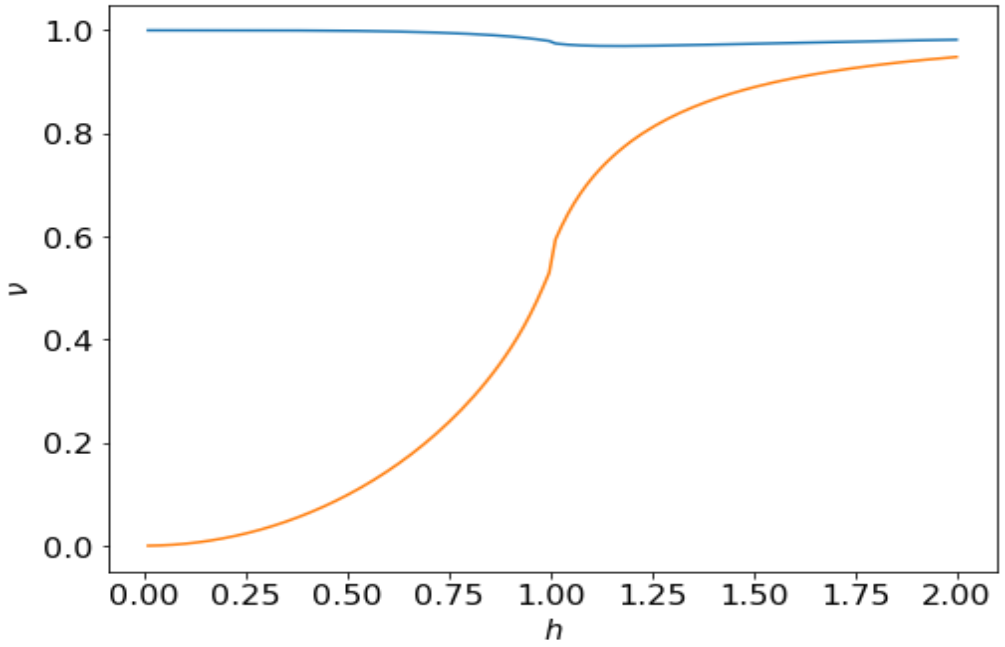
Rekombinacija rubnih stanja

- Korelacijska duljina raste s približavanjem prema faznom prijelazu
  - Više stupnjeva slobode je prepleteno
  - Rast Renyijevih entropija s malim parametrom α
- Ako Renyijeve entropije s visokim α padaju, nema lokalne konvertibilnosti
  - Dugodosežna prepletenost
- Mehanizam: rekombinacija rubnih stanja Majorana fermiona

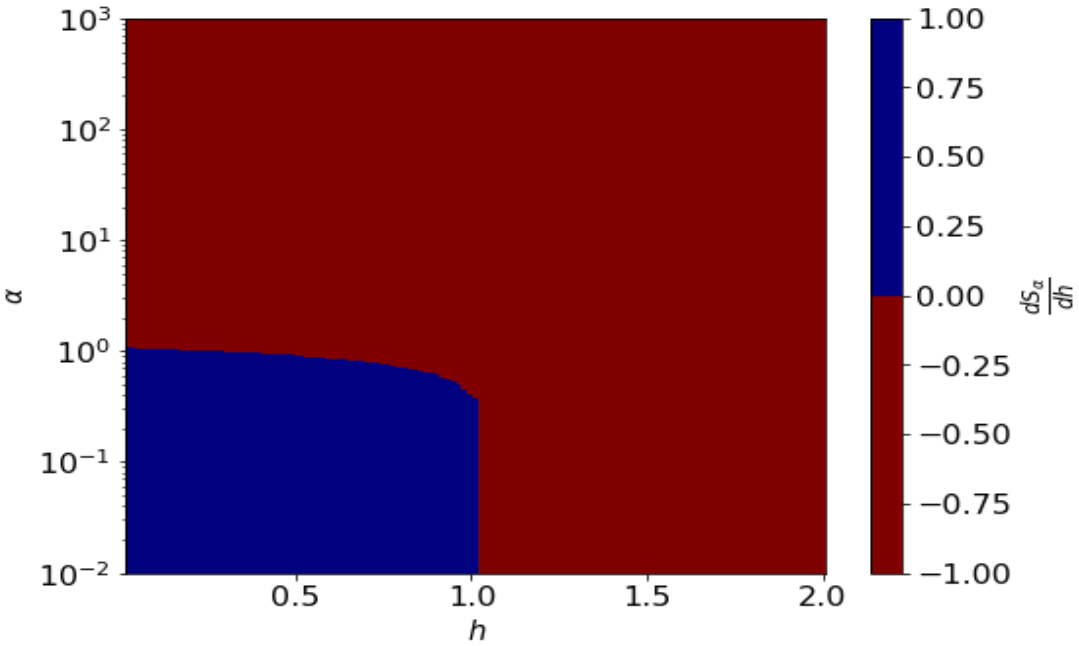


A. Y. Kitaev, Unpaired majorana fermions in quantum wires, Physics-Uspekhi 44, 131 (2001).

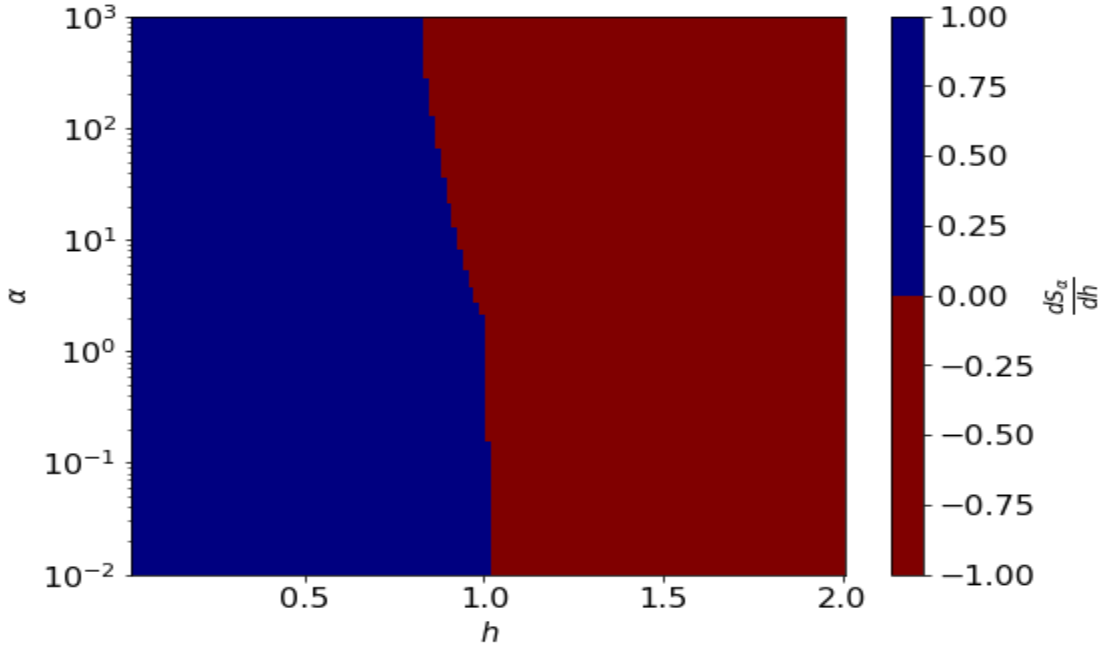
Ne-frustrirani Isingov lanac - rezultati



L=2, N=100

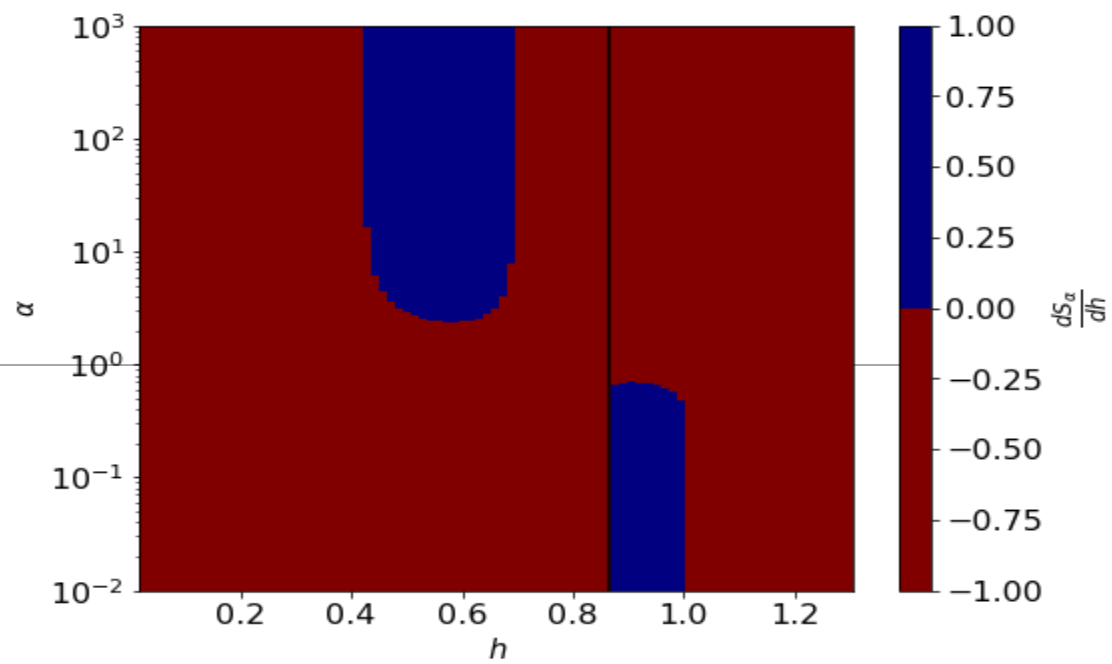


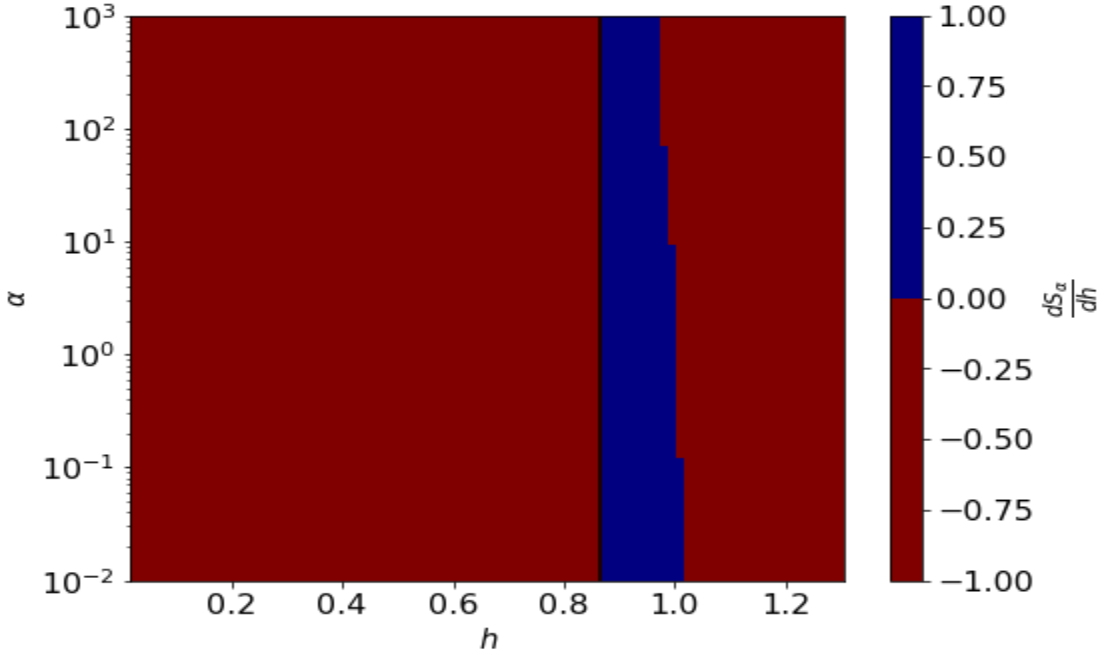
L=20, N=100

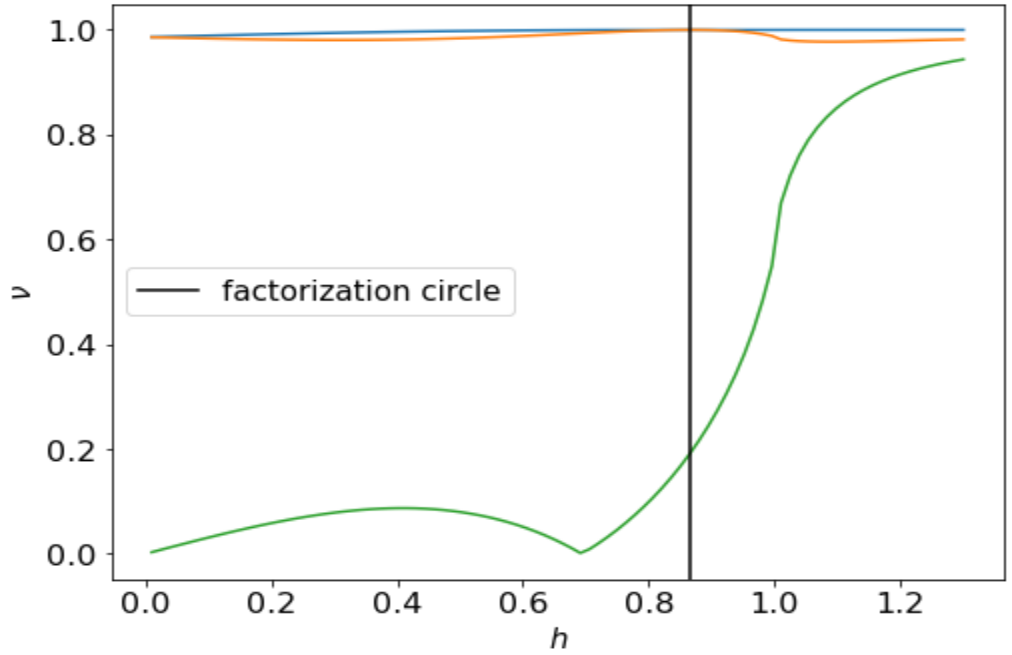


Faktorizacijska linija

- Minimum prepletenosti
- Promjena predznaka derivacija Renyijevih entropija
- Lokalna konvertibilnost nije svojstvo kvantne faze







L=50, N=151

L=3, N=101

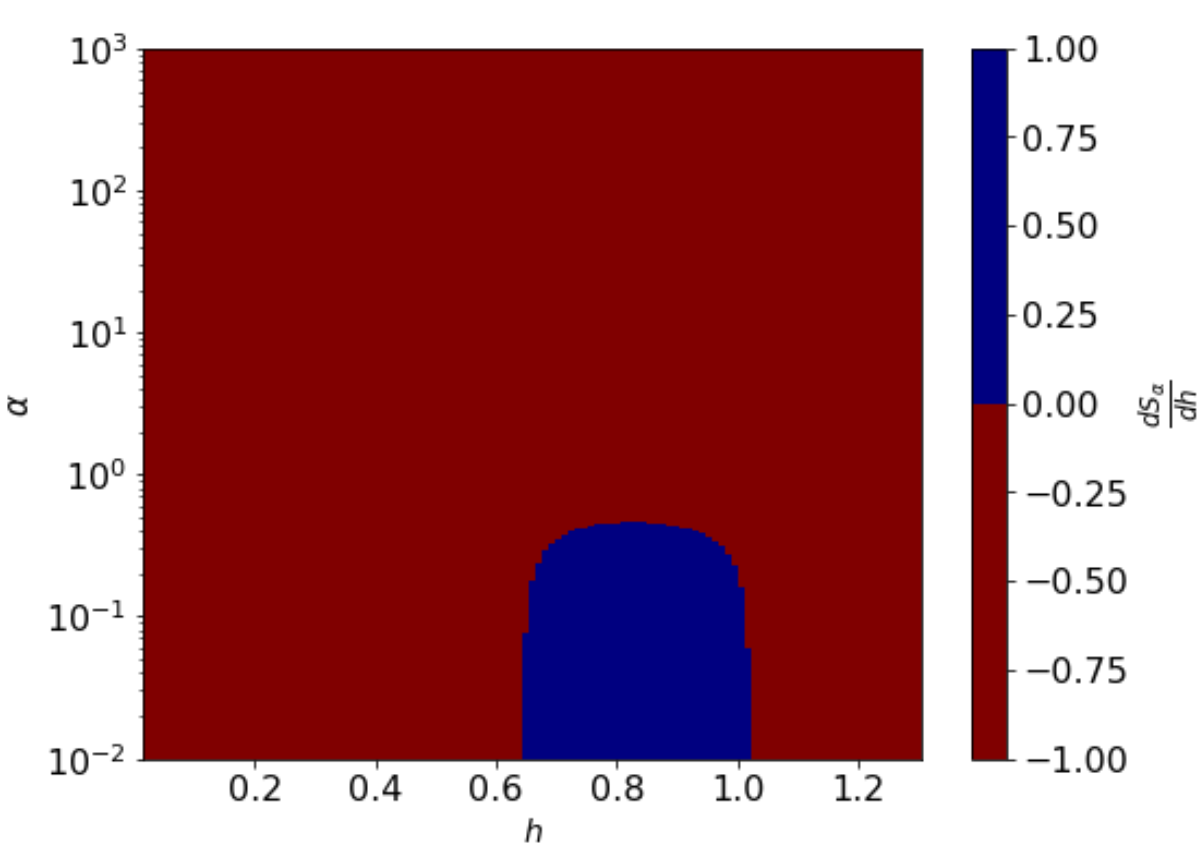
Frustrirani XY lanac

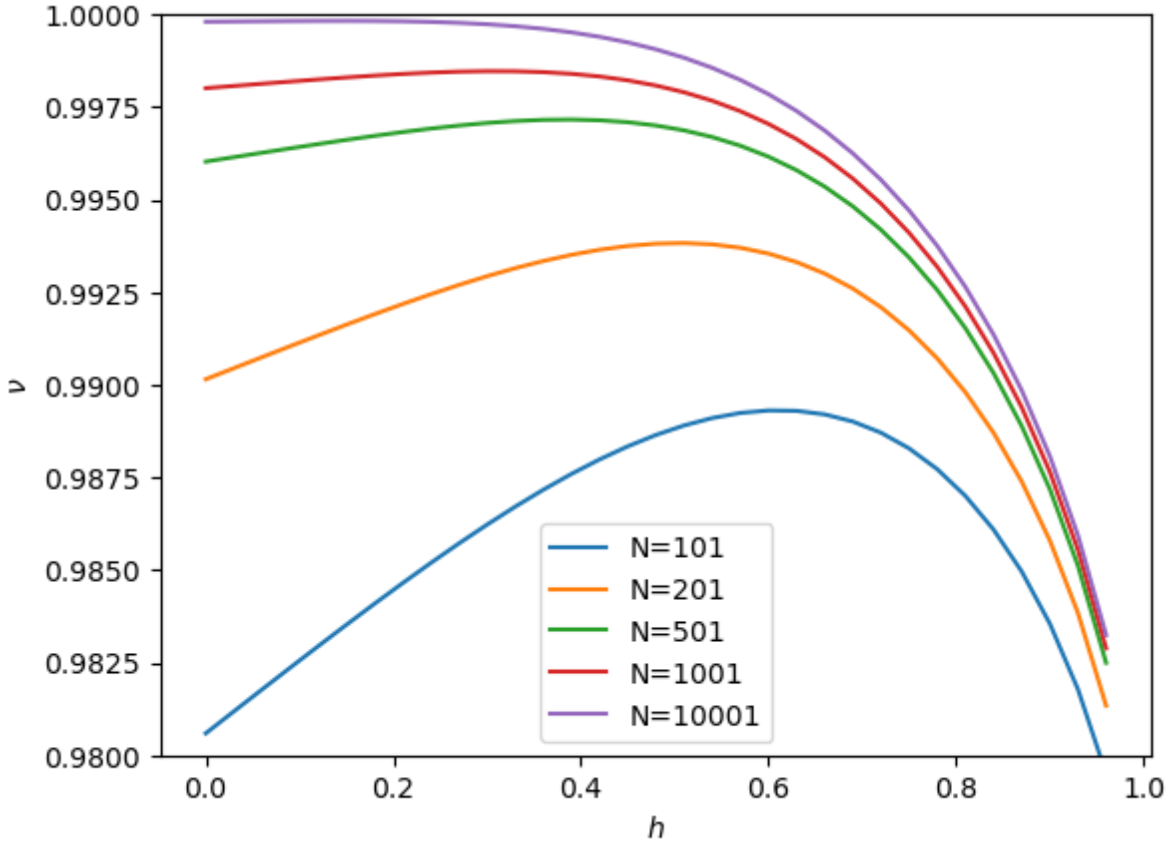
- Efektivno pobuđenje u antiferomagnetskoj fazi
- Korekcija korelacijskih funkcija

$$\langle B_j A_{j+r} \rangle = \frac{i}{N} \sum_{q \in \Gamma+} (\sin(2\theta_q) \sin(qr) + \cos(2\theta_q) \cos(qr))$$

$$\langle B_j A_{j+r} \rangle = \frac{2i(-1)^r}{N} + \frac{i}{N} \sum_{q \in \Gamma+} (\sin(2\theta_q) \sin(qr) + \cos(2\theta_q) \cos(qr))$$

- Kiralno područje ($h<1-\gamma^2$)
  - Osnovno stanje je dvostruko degenerirano, ima impuls
  - Mijenja se paritet osnovnog stanja



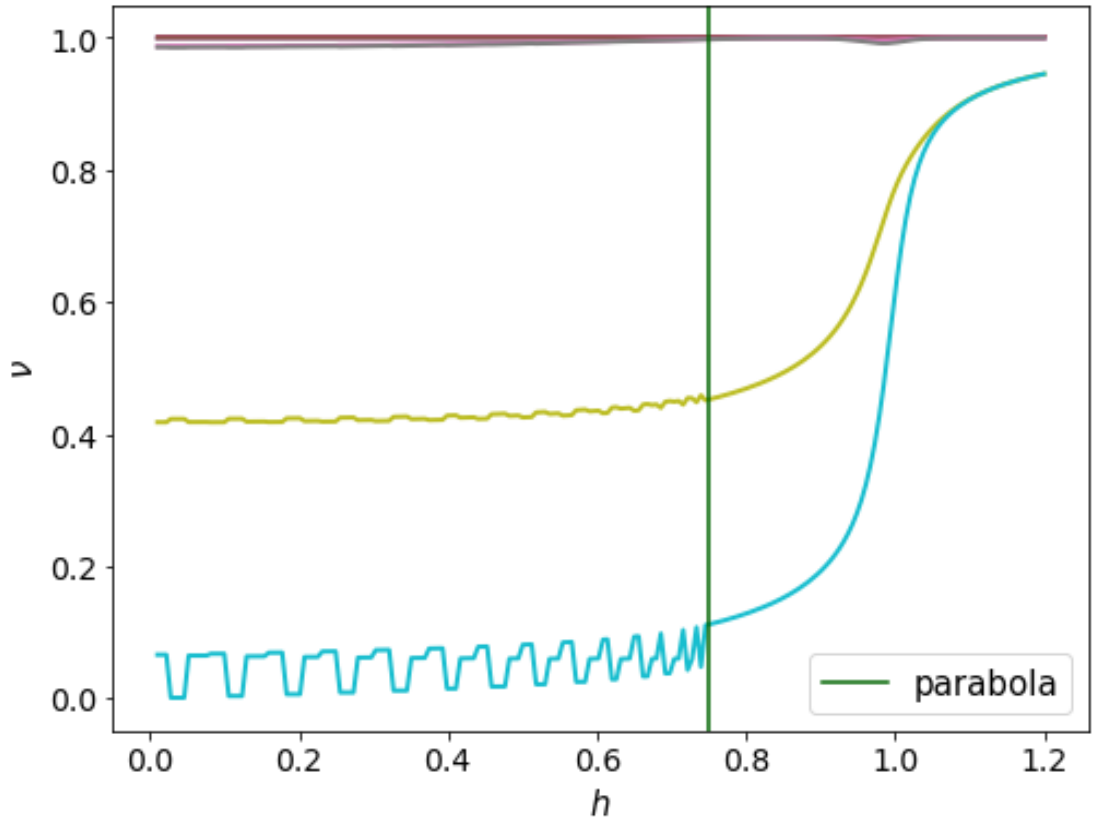


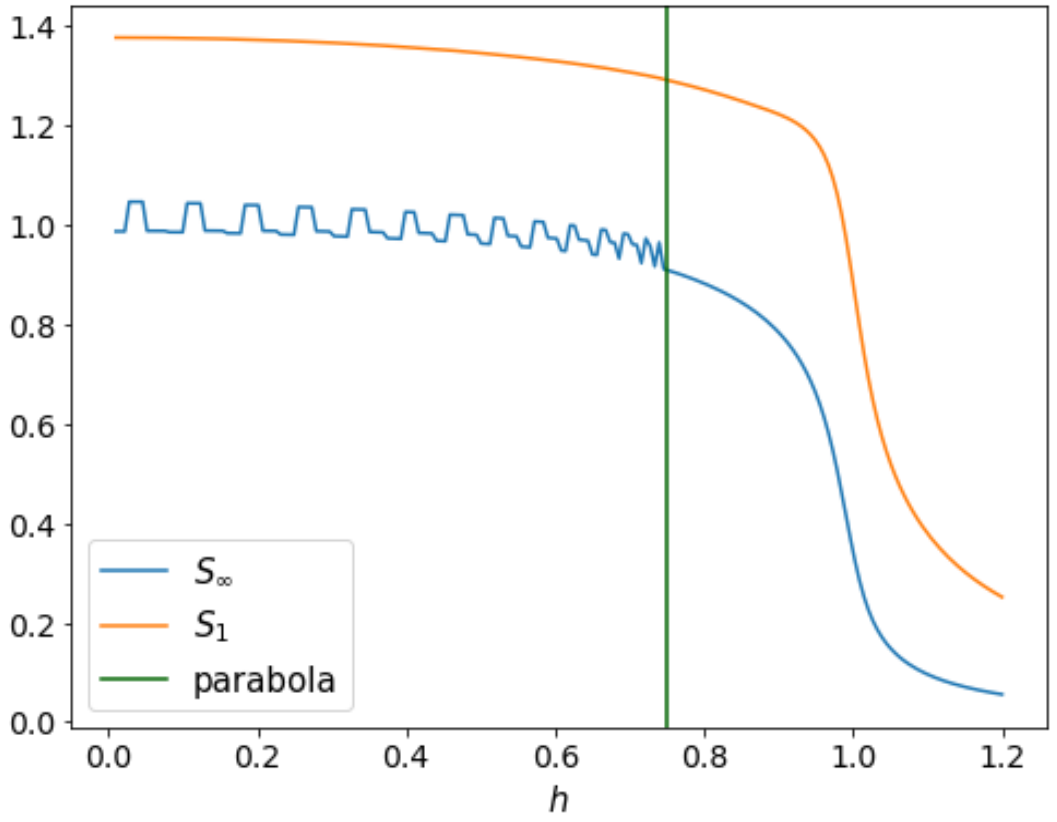
L=2, N=101

Frustrirani Isingov lanac - rezultati

Frustrirani XY lanac - rezultati

- Diskretizacija impulsa vodi na diskontinuitete u kiralnom području





L=30, N=91

W stanja

- Približna struktura osnovnog stanja u frustriranim modelima blizu klasične točke

$$|W_q\rangle = \frac{1}{\sqrt{2N}} \sum_{j=1}^{N} \exp(iqj)(|j\rangle + |j'\rangle)$$

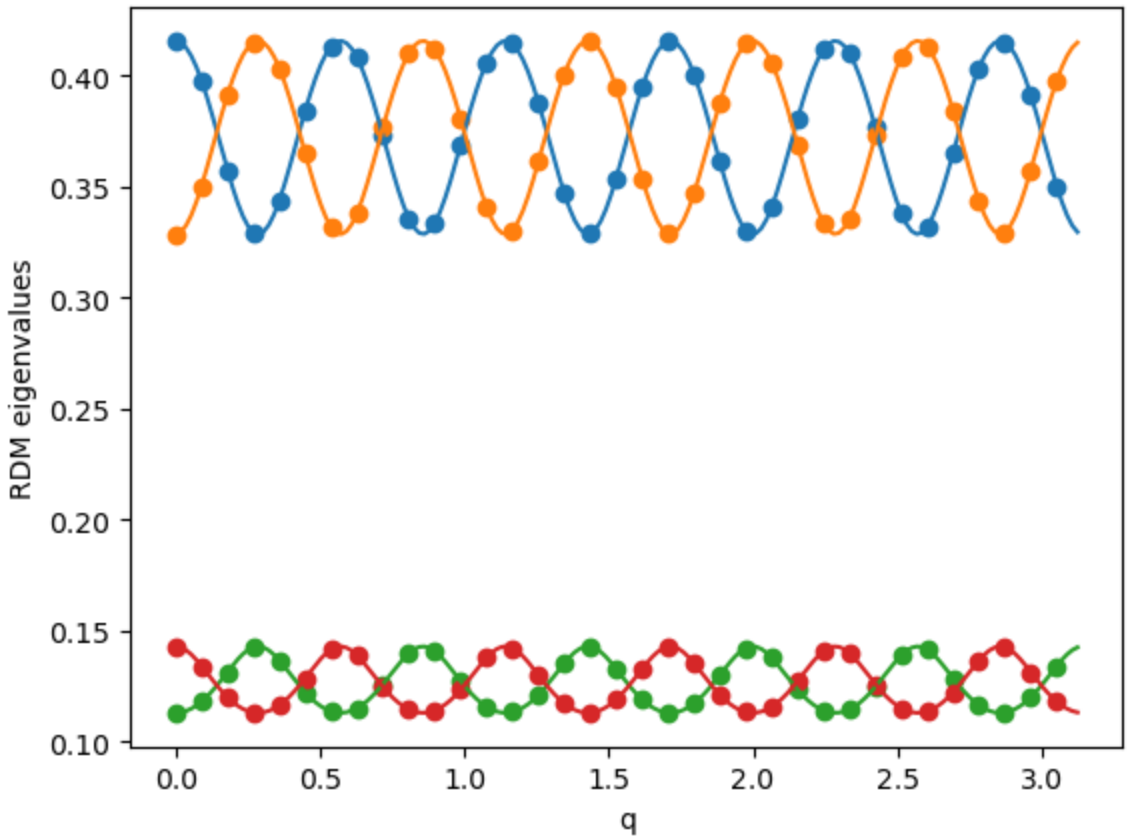
$$|j\rangle = |...+-++-+...\rangle \quad |j'\rangle = |...-+--+-...\rangle$$

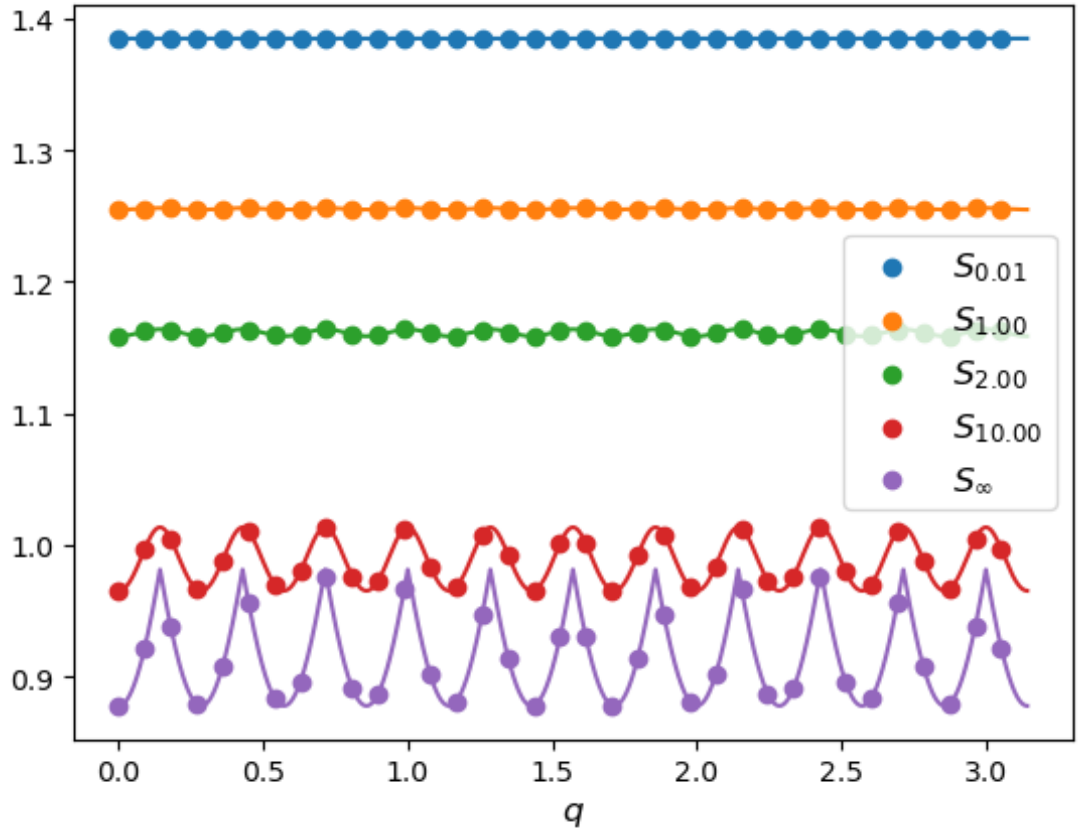
- Dozvoljeni impulsi $q \in \{\frac{\pi n}{N}\}_{n=0}^{2N-1}$

- Svojstvene vrijednosti matrice gustoće

$$\lambda_{1,...,4} = \frac{1}{4N}\left(N + 2\chi\cos(qL) \pm \sqrt{(N-2L)^2 + 4N(1+\chi\cos(qL)) - 4\sin^2(qL)}\right)$$

W stanja - oscilacije





L=11, N=35

Zaključak

- Lokalna konvertibilnost nije svojstvo kvantne faze
- Faktorizacijska linija i granica kiralnog područja imaju efekte slične faznom prijelazu
- Oscilacije u kiralnom području vode na odsustvo lokalne konvertibilnosti za bilo koji konačan N
  - Frustracija uvodi novu relevantnu skalu duljine u sustav (veličina sustava)
- Generalizirana W stanja kao struktura osnovnog stanja objašnjavaju oscilacije
  - Rezultati relevantni i za druge frustrirane sustave

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