Termodinamika prostorvremena

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Uvod

4 zakona mehanike crnih rupa (u prirodnom sustavu jedinica):

- Površinska gravitacija κ stacionarne crne rupe je konstantna.
- Za stacionarnu crnu rupu vrijedi

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Omega_H \delta J + \Phi_H \delta Q \ .$$

- Za površinu horizonta crne rupe koja nije stacionarna vrijedi

$$\delta A \geq 0 \ ,$$

dok u stacionarnom slučaju vrijedi

$$\delta A = 0 \ .$$

- Niti jedan fizikalni proces ne može svesti površinsku gravitaciju κ crne rupe na 0 u konačnom broju operacija.

$\Longrightarrow$ analogija sa zakonima termodinamike uz:

$$T = \frac{\kappa}{2\pi}\ ,\ S = \frac{A}{4}$$

Signatura $(-,+,+,+)$, sustav mjernih jedinica $c = k_B = 1$.

- Rindlerove koordinate
- Unruhova temperatura
- konstrukcija termalnog sustava
- ravnotežni tretman
- neravnotežni tretman
- $f(R)$ slučaj

Rindlerove koordinate

Za početak nalazimo putanju ubrzanog opažača, akceleracije a u recimo X smjeru. Gibanje ubrzanog opažača opisujemo pomoću tri sustava: sustav ubrzanog opažača, sustav inercijalnog opažača, sugibajući inercijski sustav.

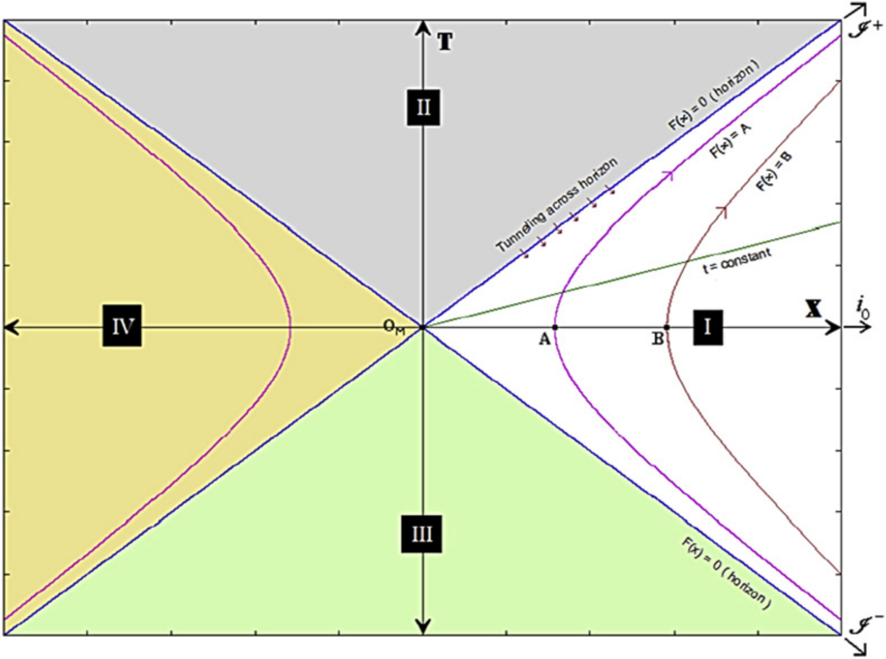
$$T(\tau) = \frac{1}{a}\sinh(a\tau)\ ,\ X(\tau) = \frac{1}{a}\cosh(a\tau)$$
$$\implies X^2 - T^2 = \frac{1}{a^2}$$

Poopćavanjem dobivamo

$$T = F(x)\sinh(\kappa t)\ ,\ X = F(x)\cosh(\kappa t)$$
$$Y = y\ ,\ Z = z\ .$$

$$\implies X^2 - T^2 = F(x)^2$$

$$\implies ds^2 = -dT^2 + dX^2 + dY^2 + dZ^2 =$$
$$= -\kappa^2 F(x)^2 dt^2 + \left(\frac{dF(x)}{dx}\right)^2 dx^2 + dy^2 + dz^2$$


T
II
IV
III
I
X
i_0
O_M
A
B
$\mathscr{I}^+$
$\mathscr{I}^-$
F(x) = 0 (horizon)
F(x) = A
F(x) = B
Tunneling across horizon
t = constant
F(x) = 0 (horizon)

Imamo 4 kauzalno razdvojena Rindlerova klina. Koristit ćemo dvije reprezentacije Rindlerove metrike:

$$F(x) = x \ , \ \text{uz } x = \frac{1}{a} \ , \tag{1}$$

$$F(x) = \frac{1 + ax}{a} \ . \tag{2}$$

$$(1) \implies ds^2 = -\kappa^2 x^2 dt^2 + dx^2 + dy^2 + dz^2$$
$$(2) \implies ds^2 = -\left(\tfrac{\kappa}{a}\right)^2 (1 + ax)^2 dt^2 + dx^2 + dy^2 + dz^2$$

Domene: $-\infty < X, T < +\infty$,

$$(1) \implies 0 \leq x < +\infty \ , -\infty < t < +\infty \ ,$$
$$(2) \implies -\tfrac{1}{a} \leq x < +\infty \ , -\infty < t < +\infty \ .$$

Unruhova temperatura

Standardni QFT izvod:

- bezmaseno skalarno (spin mu je 0) polje ϕ u $1+1$ prostorvremenu
- kvantizacija u Minkowski i Rindlerovim koordinatama
- dva skupa operatora stvaranja i poništenja $\rightsquigarrow$ Bogoljubljeve transformacije
- Minkowski i Rindlerov vakuum
- račun očekivanog broja čestica konstruiranog pomoću Rindlerovih operatora u vakuumu Minkowskog $\rightsquigarrow$ usporedba s Bose-Einsteinovom raspodjelom $\implies$ Unruhova temperatura:

$$T_U = \frac{\hbar a}{2\pi} \left(\text{u SI sustavu: } T_U = \frac{\hbar a}{2\pi c k_B} \right)$$

WKB izvod: Pokušat ćemo riješiti kovarijantnu Klein-Gordonovu jednadžbu

$$\left(\frac{1}{\sqrt{-g}}\partial_a(\sqrt{-g}g^{ab}\partial_b) - \frac{m^2}{\hbar^2}\right)\phi = 0 \ ,$$

uz (1) i (2) te $m = 0$. U oba slučaja determinanta metrike g iščezava za određeni x. Fokusirat ćemo se na slučaj (2).

$$\text{za } x' \geq -\frac{1}{2a} : \begin{cases} T = \frac{\sqrt{1+2ax'}}{a} \sinh\left(\kappa t'\right) \\ X = \frac{\sqrt{1+2ax'}}{a} \cosh\left(\kappa t'\right) \end{cases}$$

$$\text{za } x' \leq -\frac{1}{2a} : \begin{cases} T = \frac{\sqrt{|1+2ax'|}}{a} \cosh\left(\kappa t'\right) \\ X = \frac{\sqrt{|1+2ax'|}}{a} \sinh\left(\kappa t'\right) \end{cases}$$

Domene: $-\infty < x', t' < +\infty$.

$$\implies ds^2 = -\left(\frac{\kappa}{a}\right)^2 (1+2ax')dt'^2 + (1+2ax')^{-1}dx'^2$$

$$\implies g = -\left(\frac{\kappa}{a}\right)^2 \neq 0 \ \forall \ x', t'$$

- Schwarzschildova metrika u sfernim koordinatama:

$$ds^2 = -\left(1-\frac{2GM}{r}\right)dt^2 + \left(1-\frac{2GM}{r}\right)^{-1}dr^2 + \underbrace{\cancel{r^2 d\Omega^2}}_{\text{efektivno } y \text{ i } z}$$

$\implies$ horizont: $x' = -\frac{1}{2a}$
$\implies$ tuneliranje između klinova **I** i **II** $\rightsquigarrow T^2 - X^2 = \frac{1}{a^2}$
$\implies \phi(x) = \phi_0 e^{\frac{i}{\hbar}S(x)}$ uvrštavamo u kovarijantnu K-G jednadžbu (dijagonalna g_{ab} + harmoničko baždarenje: $\Gamma^b_{ad}g^{ad} = 0$ + poluklasični limes $\hbar \to 0$) te dobivamo

$$g^{ab}(\partial_a S(x))(\partial_b S(x)) = 0$$

- očuvanje energije: $S(x) = S(t, \vec{x}) = -Et + S_0(\vec{x})$

- $\mathbf{T} \sim e^{-\frac{1}{\hbar}\mathfrak{Im}\left(2\int p(x)\,dx - E\Delta t\right)}$
- nakon procesa termalizacije: $\mathbf{T} \sim e^{-\frac{E}{T}} \rightsquigarrow$ Maxwell-Boltzmann
- $S_0(x) = \int p(x)dx$

$\implies T = \frac{E\hbar}{\mathfrak{Im}(2S_0(x) - E\Delta t)}\ ,\ \Delta t = t^+ + t^-$

$\implies -\left(\frac{a}{\kappa}\right)^2 \frac{E^2}{1+2ax'} + (1+2ax')(\partial_{x'} S_0(x'))^2 = 0$

$\overset{u=1+2ax'}{\implies} S_0 = \frac{E}{2\kappa}\int_{-\infty}^{+\infty}\frac{du}{u}$

Sokhotski-Plemeljov teorem

Neka je $f : \mathbb{R} \to \mathbb{C}$ neprekidna funkcija i $a < 0 < b$. Tada je

$$\lim_{\epsilon \to 0^+} \int_a^b \frac{f(x)}{x \pm i\epsilon}\, dx = \mp i\pi f(0) + P.V. \int_a^b \frac{f(x)}{x}\, dx\ ,$$

gdje $P.V.$ označava Cauchyjevu glavnu vrijednost, koja je definirana kao

$$P.V. \int_a^b f(x)\, dx = \lim_{\epsilon \to 0^+} \left(\int_a^{c-\epsilon} f(x)\, dx + \int_{c+\epsilon}^b f(x)\, dx \right)\ ,$$

gdje vrijedi $a < c < b$ i $f(x)$ ima singularitet u c.

$$\overset{\Im\mathfrak{m}(S_0)>0}{\Longrightarrow} S_0 = i\frac{E\pi}{2\kappa} + \frac{E}{2\kappa}\underbrace{P.V.\int_{-\infty}^{+\infty}\frac{du}{u}}_{=0} \Longrightarrow S_0 = i\frac{E\pi}{2\kappa}$$

$$\mathbf{Y} = 1 + 2ax' \Longrightarrow \begin{cases} x' \leq -\frac{1}{2a} \Longrightarrow \mathbf{Y} \leq 0 \\ x' \geq -\frac{1}{2a} \Longrightarrow \mathbf{Y} \geq 0 \end{cases}$$

$$\underbrace{\sqrt{|\mathbf{Y}|} = \sqrt{-\mathbf{Y}}}_{\mathbf{Y}\leq 0} \longrightarrow \underbrace{\sqrt{(-)(-)(-\mathbf{Y})} = i\sqrt{\mathbf{Y}}}_{\mathbf{Y}\geq 0}$$

$$\underbrace{\sqrt{\mathbf{Y}}}_{\mathbf{Y}\geq 0} \longrightarrow \underbrace{\sqrt{(-)(-\mathbf{Y})} = i\sqrt{-\mathbf{Y}} = i\sqrt{|\mathbf{Y}|}}_{\mathbf{Y}\leq 0}$$

$\Longrightarrow$ zbog konzistentnosti transformacija između (X, T) i (x', t'):

$$t' \longrightarrow t' - \frac{i\pi}{2\kappa}$$

$$\implies t'^{\pm} = -\frac{i\pi}{2\kappa} \implies \Delta t' = t'^{+} + t'^{-} = -\frac{i\pi}{\kappa}$$
$$\implies T = \frac{\hbar\kappa}{2\pi}$$

Tolmanov zakon

Ako imamo neki medij na temperaturi T (u našem kontekstu je preciznije reći da je to temperatura koju mjeri opažač akceleracije $a = \kappa$) i neku statičnu metriku (metrika je statična ako je stacionarna $(g_{00}(x) = g_{00}(\vec{x}))$ te ako vrijedi $g_{0i} = 0$ i $g_{ij}(x) = g_{ij}(\vec{x})$ (gdje je $i, j = 1, 2, 3$)), onda je temperatura koju mjeri neki opažač ($T_{op}(\vec{x})$) jednaka $T_{op}(\vec{x}) = \frac{T}{\sqrt{-g_{00}(\vec{x})}}$.

$\implies$ uz reprezentaciju $F(x') = x' = \frac{1}{a}$ imamo $\sqrt{-g_{00}} = \frac{\kappa}{a}$

$$\implies T_U = \frac{\hbar\cancel{\kappa}}{2\pi}\frac{1}{\frac{\cancel{\kappa}}{a}} \implies T_U = \frac{\hbar a}{2\pi}$$

Formaliziranje WKB izvoda:

- $3+1$ prostorvrijeme
- bezmaseno skalarno polje: kovarijantna Klein-Gordonova jednadžba $\rightsquigarrow$ modificirane Besselove funkcije
- razvoj u blizini horizonta $(F(x') = 0)$
- matrica gustoće mnogočestične valne funkcije $\implies$ parcijalni trag $\rightsquigarrow$ micanje izlaznih valnih funkcija koje ne doprinose termalnom spektru
- očekivana vrijednost operatora broja čestica $\rightsquigarrow$ Bose-Einsteinova raspodjela $\implies$ Unruhova temperatura
- bezmaseno fermionsko polje $\left(\text{spin mu je } \frac{1}{2}\right)$: kovarijantna Diracova jednadžba

$$(i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m)\Psi = 0$$

  $\rightsquigarrow$ modificirane Besselove funkcije
- razvoj u blizini horizonta $(F(x') = 0)$
- matrica gustoće mnogočestične valne funkcije $\implies$ parcijalni trag
- očekivana vrijednost operatora broja čestica $\rightsquigarrow$ Fermi-Diracova raspodjela $\implies$ Unruhova temperatura

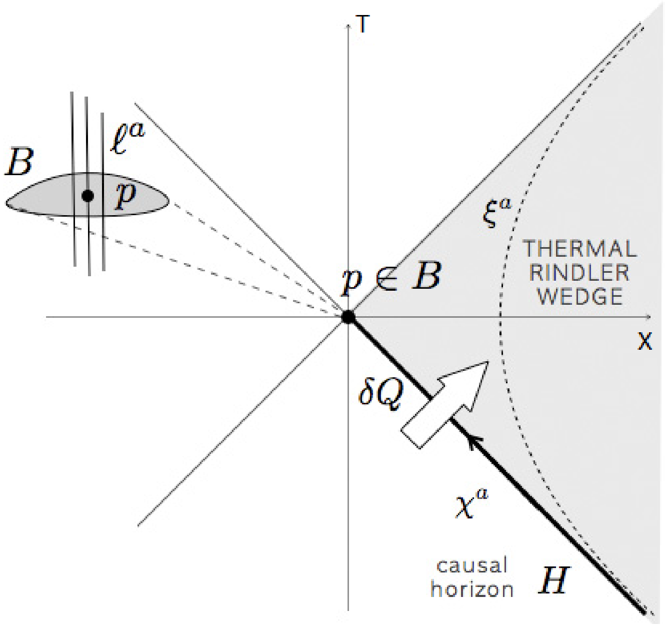
Konstrukcija termalnog sustava

$\underline{F(x) = x:}$

$$\partial_t = \frac{\partial T}{\partial t}\partial_T + \frac{\partial X}{\partial t}\partial_X + \frac{\partial Y}{\partial t}\partial_Y + \frac{\partial Z}{\partial t}\partial_Z =$$
$$= \kappa\left(x\cosh(\kappa t)\partial_T + x\sinh(\kappa t)\partial_X\right) + 0 + 0 =$$
$$= \kappa\left(X\partial_T + T\partial_X\right)$$
$$\implies \partial_t = \kappa\underbrace{\left(X\partial_T + T\partial_X\right)}_{\equiv\chi^a}$$

$$\partial_T = \begin{pmatrix}1\\0\\0\\0\end{pmatrix},\ \partial_X = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \implies \chi^a = \begin{pmatrix}X\\T\\0\\0\end{pmatrix}$$

$$\chi_a\chi^a = g_{ab}\chi^a\chi^b = g_{00}\chi^0\chi^0 + g_{11}\chi^1\chi^1 = -X^2 + T^2 = 0 \implies X = \pm T$$


T
ℓ^a
B
p
ξ^a
THERMAL
RINDLER
WEDGE
$p \in B$
X
δQ
χ^a
causal
horizon
H

Teorem

Ako su generatori Killingovog horizonta geodetski kompletni u prošlost, uz neiščezavajuću površinsku gravitaciju, onda Killingov horizont sadrži $(D-2)$-dimenzionalnu (u našem slučaju dvodimenzionalnu) prostornoliku plohu B na kojoj Killingovo vektorsko polje χ^a iščezava. B nazivamo bifurkacijskom plohom.

$$l^a l^b{}_{;a} = 0$$

$$\chi^a = -\kappa\lambda l^a, \ \lambda < 0$$

$$\implies \chi^a \chi^b{}_{;a} = \kappa \chi^b$$

$$\implies \lambda = -e^{-\kappa v}$$

$$T = T_U \sqrt{-g_{00}} = \frac{\hbar \cancel{a}}{2\pi} \frac{\kappa}{\cancel{a}} = \frac{\hbar\kappa}{2\pi} \implies T = \frac{\hbar\kappa}{2\pi}$$

$$\delta Q = \int_H T_{ab}\chi^a d\Sigma^b \ , \ d\Sigma^b = l^b \widetilde{\epsilon} d\lambda \ , \ \int_H \widetilde{\epsilon} d\lambda(...) = \int d\lambda \int_{H(\lambda)} \widetilde{\epsilon}(...)$$

- afino parametrizirana Raychaudhurijeva jednadžba za svjetlosne geodezike:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^{ab}\sigma_{ab} + \underbrace{\omega^{ab}\omega_{ab}}_{=0 \text{ (po konstrukciji)}} - R_{ab} l^a l^b$$

$$\theta = l^a{}_{;a} \ , \ \theta = \frac{1}{\widetilde{\epsilon}}\frac{d\widetilde{\epsilon}}{d\lambda} \equiv \frac{1}{\widetilde{\epsilon}}\dot{\widetilde{\epsilon}}$$

$$\theta(v) = -\kappa\lambda\theta(\lambda) \ , \ \sigma_{ab}(v)\sigma^{ab}(v) = (-\kappa\lambda)^2 \sigma_{ab}(\lambda)\sigma^{ab}(\lambda)$$

Ravnotežni tretman

- jaki princip ekvivalencije: $dS = \alpha\delta A = \alpha\widetilde{\epsilon}$, $\alpha = konst.$
- Clausiusova relacija: $\delta Q = TdS$

$$\delta Q = T\alpha\widetilde{\epsilon} = T\alpha \int_H d\widetilde{\epsilon} = T\alpha \int_H \theta(\lambda)\widetilde{\epsilon} d\lambda$$

$$\theta(\lambda) \approx \theta(\lambda)_p + \lambda \frac{d\theta(\lambda)}{d\lambda}\bigg|_p$$

$$\delta Q = T\alpha \int_H \widetilde{\epsilon} d\lambda \left(\theta(\lambda)_p + \lambda \left(-\frac{1}{2}\theta(\lambda)^2 - \sigma^{ab}(\lambda)\sigma_{ab}(\lambda) - R_{ab}l^a l^b \right) \bigg|_p \right)$$

$$\delta Q = \int_H T_{ab}\chi^a d\Sigma^b = \int_H T_{ab}(-\kappa\lambda) l^a l^b \widetilde{\epsilon} d\lambda$$

$$\implies \lambda = 0 \implies \delta Q = 0 \implies \theta(\lambda)_p = 0$$

$$\implies \int_H \widetilde{\epsilon} d\lambda (-\kappa\lambda) T_{ab} l^a l^b = T\alpha \int_H \widetilde{\epsilon} d\lambda (-\lambda)(\sigma^{ab}(\lambda)\sigma_{ab}(\lambda) + R_{ab}l^a l^b)\bigg|_p$$

- pretpostavka: $\sigma_{ab}(\lambda)_p = 0$

$$\Longrightarrow \int_H \widetilde{\epsilon} d\lambda \kappa \lambda T_{ab} l^a l^b = \frac{\hbar\kappa}{2\pi}\alpha \int_H \widetilde{\epsilon} d\lambda \lambda R_{ab} l^a l^b$$

$$\Longrightarrow \frac{2\pi}{\hbar\alpha} T_{ab} l^a l^b = R_{ab} l^a l^b$$

- l^a je svjetlosnog tipa: $l_a l^a = g_{ab} l^a l^b = 0$

$$\Longrightarrow \frac{2\pi}{\hbar\alpha} T_{ab} = R_{ab} + \mathbf{\Psi} g_{ab}$$

- $\nabla^b T_{ab} = 0$, $\nabla^b R_{ab} = \frac{1}{2}\nabla_a R$

$$\Longrightarrow \mathbf{\Psi} = -\frac{R}{2} + \Lambda$$

$$\Longrightarrow R_{ab} - \frac{1}{2} g_{ab} R + \Lambda g_{ab} = \frac{2\pi}{\hbar\alpha} T_{ab}$$

- $\mathcal{S}_{EH} = \int \left(\frac{1}{16\pi G}(R - 2\Lambda) + \mathcal{L}_M \right) \sqrt{-g}\, d^4x$, $T^{ab} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{ab}}$

$$\implies \delta \mathcal{S}_{EH} = 0 \implies R_{ab} - \frac{1}{2} g_{ab} R + \Lambda g_{ab} = 8\pi G T_{ab}$$

$$R_{ab} - \frac{1}{2} g_{ab} R + \Lambda g_{ab} = \frac{2\pi}{\hbar\alpha} T_{ab}$$
$$R_{ab} - \frac{1}{2} g_{ab} R + \Lambda g_{ab} = 8\pi G T_{ab}$$

$$\implies \frac{2\pi}{\hbar\alpha} = 8\pi G \implies \alpha = \frac{1}{4\hbar G}$$

$$\implies \frac{1}{\sqrt{\alpha}} = 2\sqrt{\hbar G} \rightsquigarrow \text{UV regulator: } l_{UV} \sim l_P = \sqrt{\hbar G}$$

Neravnotežni tretman

$$\theta(v) = \kappa e^{-\kappa v}\theta(\lambda)\ ,\ \sigma_{ab}(v)\sigma^{ab}(v) = (\kappa e^{-\kappa v})^2\sigma_{ab}(\lambda)\sigma^{ab}(\lambda)$$
$$\implies \theta(\lambda)_p \neq 0\ ,\ \sigma_{ab}(\lambda)_p \neq 0$$
$$\implies dS \geq \frac{\delta Q}{T} \implies \text{zakon ravnoteže entropije: } dS = \frac{\delta Q}{T} + \underbrace{dS_i}_{\geq 0}$$

Relativistička disipativna hidromehanika:

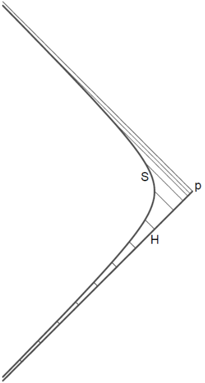
- idealni fluid: $T^{ab} = (\rho + p)u^a u^b + pg^{ab}$
- smetnja:
  $\Pi^{ab} = -2\eta\sigma^{ab} - \zeta\theta h^{ab} \rightsquigarrow$ ovisnost o prvim derivacijama u^a

$$\sigma^{ab} = \frac{1}{2}(u^a{}_{;c}h^{cb} + u^b{}_{;c}h^{ca}) - \frac{\theta}{3}h^{ab}$$
$$h^{ab} = g^{ab} + u^a u^b$$

- Landauovo baždarenje: $\Pi^{ab}u_b = 0$

$$\implies \nabla_a j_s^a = \nabla_a (s u^a) = \frac{2\eta}{T}\sigma^{ab}\sigma_{ab} + \frac{\zeta}{T}\theta^2$$

$$\implies dS_{vis} = \int \widetilde{\epsilon} dv \left(\frac{2\eta}{T}\sigma^{ab}\sigma_{ab} + \frac{\zeta}{T}\theta^2 \right)$$



- $\frac{\delta Q}{T} = -\frac{\kappa}{T}\int_H \lambda T_{ab} l^a l^b \widetilde{\epsilon} d\lambda = \alpha \int_H \widetilde{\epsilon} d\lambda \left(\theta(\lambda) - \lambda R_{ab} l^a l^b\right) \Big|_p$

  $\implies \theta(\lambda)_p = 0 \implies$ jednadžbe polja za OTR

- $dS_i = -\alpha \int_H \lambda \left(\frac{1}{2} \underbrace{\theta(\lambda)^2}_{=0} + \sigma^{ab}(\lambda)\sigma_{ab}(\lambda) \right) \Bigg|_p \widetilde{\epsilon} d\lambda$

$$\implies dS_i = -\alpha \int_H \widetilde{\epsilon} d\lambda \lambda \sigma^{ab}(\lambda)\sigma_{ab}(\lambda) \Big|_p$$

$$\implies dS_i = \frac{\alpha}{\kappa} \int_H \widetilde{\epsilon} dv \sigma^{ab}(v)\sigma_{ab}(v) \Big|_p$$

$$\implies \frac{2\eta}{T} = \frac{\alpha}{\kappa} \implies \eta = \frac{T\alpha}{2\kappa} = \frac{\alpha}{2\kappa}\frac{\hbar\kappa}{2\pi} = \frac{\alpha\hbar}{4\pi}$$

$$\implies \frac{\eta}{\alpha} = \frac{\hbar}{4\pi} \rightsquigarrow \text{AdS/CFT}$$

$$\implies T dS_i = \frac{1}{8\pi G} \int_H \widetilde{\epsilon} dv \sigma^{ab}(v)\sigma_{ab}(v) \Big|_p \rightsquigarrow \text{Hartle-Hawking}$$

$f(R)$ slučaj

- $\mathcal{F}(R) = \frac{df(R)}{dR}$, OTR slučaj: $f = R - 2\Lambda \implies \mathcal{F} = 1$, $\dot{\mathcal{F}} = 0$
- Einsteinov princip ekvivalencije: $dS = \beta \mathcal{F} \widetilde{\epsilon}$, $\beta = konst.$

$$\implies dS = \beta \int_H \widetilde{\epsilon} d\lambda \underbrace{(\dot{\mathcal{F}} + \mathcal{F}\theta(\lambda))}_{=\widetilde{\theta}(\lambda)}$$

- $\widetilde{\theta}(\lambda)_p = 0 \implies \theta(\lambda)_p = -\frac{\dot{\mathcal{F}}}{\mathcal{F}}$

$$\widetilde{\theta}(\lambda) \approx \widetilde{\theta}(\lambda)_p + \lambda \frac{d\widetilde{\theta}(\lambda)}{d\lambda}\bigg|_p$$

$$\implies dS = \beta \int_H \widetilde{\epsilon} d\lambda \lambda \bigg((\mathcal{F}_{;ab} - \mathcal{F} R_{ab}) l^a l^b - \frac{3}{2}\mathcal{F}\theta(\lambda)^2 - \mathcal{F}\sigma^{ab}(\lambda)\sigma_{ab}(\lambda) \bigg)\bigg|_p$$

- $\frac{\delta Q}{T} = \beta \int_H \widetilde{\epsilon} d\lambda \lambda (\mathcal{F}_{;ab} - \mathcal{F} R_{ab}) l^a l^b$

$$\Longrightarrow \frac{2\pi}{\hbar\beta} T_{ab} = \mathcal{F} R_{ab} - \mathcal{F}_{;ab} + \widetilde{\mathbf{\Psi}} g_{ab}$$

$$\Longrightarrow \widetilde{\mathbf{\Psi}} = \Box \mathcal{F} - \frac{f}{2}$$

$$\Longrightarrow \mathcal{F} R_{ab} - \mathcal{F}_{;ab} + \left(\Box \mathcal{F} - \frac{f}{2} \right) g_{ab} = \frac{2\pi}{\hbar\beta} T_{ab}$$

- $\mathcal{S}_{f(R)} = \int \left(\frac{1}{16\pi G} f(R) + \mathcal{L}_M \right) \sqrt{-g}\, d^4 x$, $T^{ab} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g_{ab}}$

$$\Longrightarrow \delta \mathcal{S}_{f(R)} = 0 \Longrightarrow \mathcal{F} R_{ab} - \mathcal{F}_{;ab} + \left(\Box \mathcal{F} - \frac{f}{2} \right) g_{ab} = 8\pi G T_{ab}$$

$$\Longrightarrow \frac{2\pi}{\hbar\beta} = 8\pi G \Longrightarrow \beta = \frac{1}{4\hbar G} \Longrightarrow \beta = \alpha$$

- $dS_i = -\beta \int_H \widetilde{\epsilon} d\lambda \lambda \mathcal{F} \left(\frac{3}{2}\theta(\lambda)^2 + \sigma^{ab}(\lambda)\sigma_{ab}(\lambda) \right) \Big|_p$

$$\implies dS_i = \frac{\beta}{\kappa} \int_H \widetilde{\epsilon} dv \mathcal{F} \left(\frac{3}{2}\theta(v)^2 + \sigma^{ab}(v)\sigma_{ab}(v) \right) \Big|_p$$

$$\implies \frac{\eta}{\alpha} = \frac{\hbar \mathcal{F}}{4\pi}$$

$$\implies \frac{\zeta}{\alpha} = \frac{3\hbar \mathcal{F}}{4\pi}$$

Zaključak

- proporcionalnost entropije i površine lokalnog horizonta
- odabir principa ekvivalencije $\implies$ odgovarajuće jednadžbe polja
- jednadžbe polja kao jednadžbe stanja
- neravnotežni doprinosi $\rightsquigarrow$ poznate interpretacije
- L. Boltzmann: temperatura $\implies$ mikrostruktura
- statistička/termodinamička priroda gravitacije $\rightsquigarrow$ Sir A. S. Eddington
- stupnjevi slobode gravitacije koji se ne mogu obuhvatiti jednadžbama polja $\rightsquigarrow$ mikrostruktura prostorvremena

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