N-body simulations in cosmology

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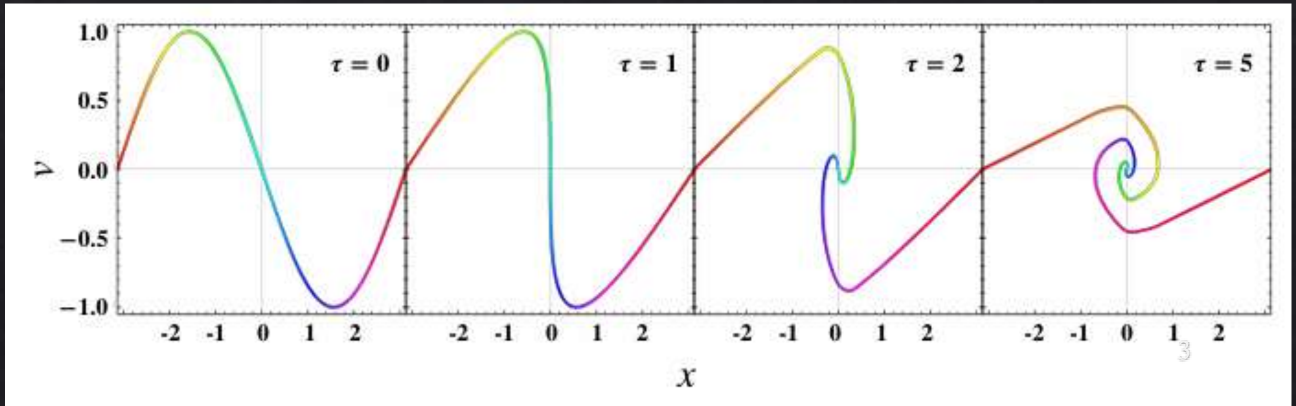
Introduction

- theoretical model of the universe
- dark energy represented by a cosmological constant Λ
- dark matter as a self-gravitating, pressureless (cold) fluid
- ΛCDM cosmological model describes the evolution of the universe
- the emergence of highly nonlinear structures due to gravitational self-interaction
- easiest way to predict evolution is through numerical simulations
- motion of matter described using N particles
- two parts of the simulation:
  - calculation of initial conditions
  - particle position and momentum updates

Calculation of initial conditions

- evolution of the dark matter distribution *f = f (**x**, **p**, t)* in phase space
- described by the Vlasov-Poisson system of equations
- due to the topology of the distribution and the Hamiltonian nature of the system, the three-dimensional hypersurface remains continuous with no self-intersections
- phenomenon of hypersurface folding known as shell-crossing
- infinite densities of matter when hypersurface folds

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{ma^2} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}} f - m(\boldsymbol{\nabla}_{\boldsymbol{x}}\phi) \cdot \boldsymbol{\nabla}_{\boldsymbol{p}} f = 0$$
$$\boldsymbol{\nabla}^2_{\boldsymbol{x}}\phi = 4\pi G\bar{\rho}(t)a^2(t)\,\delta(\boldsymbol{x}, t)$$



toy example in 1D

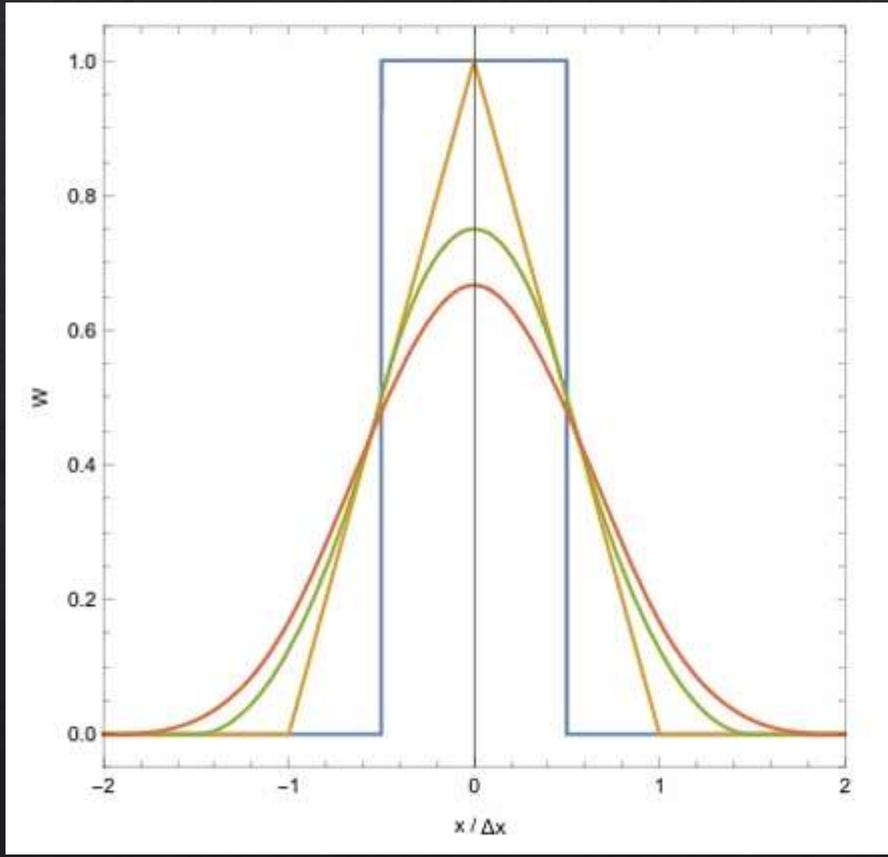
Solving the Vlasov-Poisson equations

- before shell crossing: using Lagrangian perturbation theory (LPT)
- introducing Lagrangian mapping: $\boldsymbol{q} \rightarrow \boldsymbol{x}(\boldsymbol{q}, t) = \boldsymbol{q} + \boldsymbol{\psi}(\boldsymbol{q}, t)$
- solving by expanding the Lagrangian displacement field in a series
- obtaining the truncated solution of *n*-th order (nLPT)
- using this solution for initial conditions
- obtaining *N* particles with corrected positions and velocities from a uniform grid
- a later initial time leads to larger errors

$$\psi(\boldsymbol{q}, t) = \sum_{n=1}^{\infty} \psi^{(n)}(\boldsymbol{q}) D_+^n$$

Simulation of particle interactions

- in particle-mesh simulations
- resolution defined by the number of particles and grid points
- the force on particles is calculated using densities computed at the grid points
- densities are calculated using the mass assignment scheme
- integrated in multiple time steps
- earlier initial times lead to larger errors

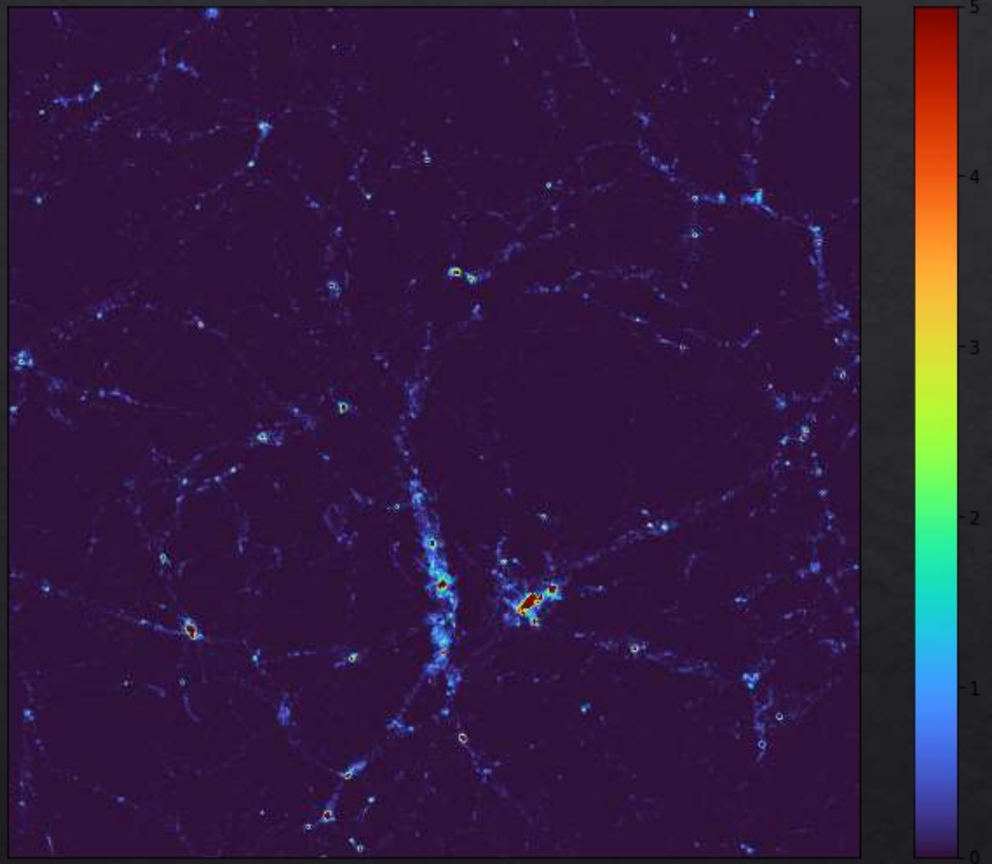


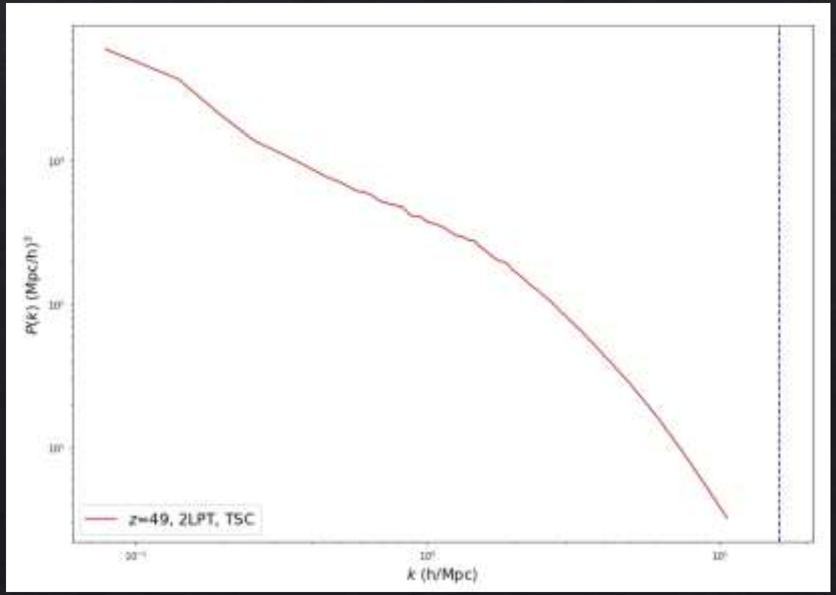
Problems

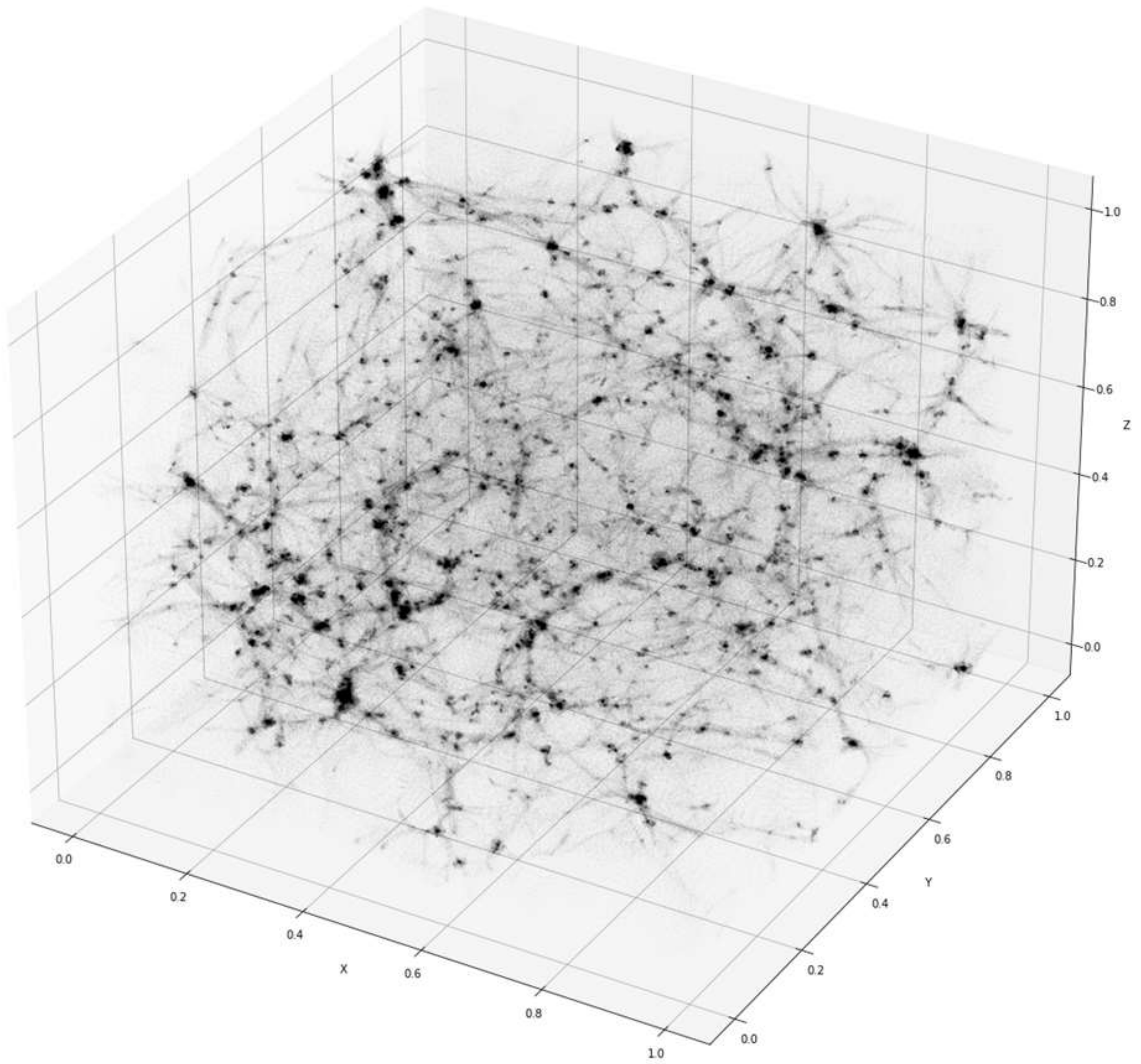
- what is a good time to initialise the simulations using LPT
- selection of the initial time defined by the redshift: $z = 1/a(t) - 1$
- larger z corresponds to a smaller universe, or earlier times, while $z = 0$ corresponds to the present moment
- importance of using the number of terms before truncating the order in perturbative calculations
- impact of the mass assignment scheme on the results
- impact of the number of particles and grid points on the results

Results of a single simulation

- PYSCO code used to obtain the results
- the obtained results are the positions and velocities of particles at the final time, as well as the matter power spectra
- from the positions we obtain xy-slices
- matter power spectrum represents a two-point statistic and is derived as the Fourier transform of the mass density autocorrelation function



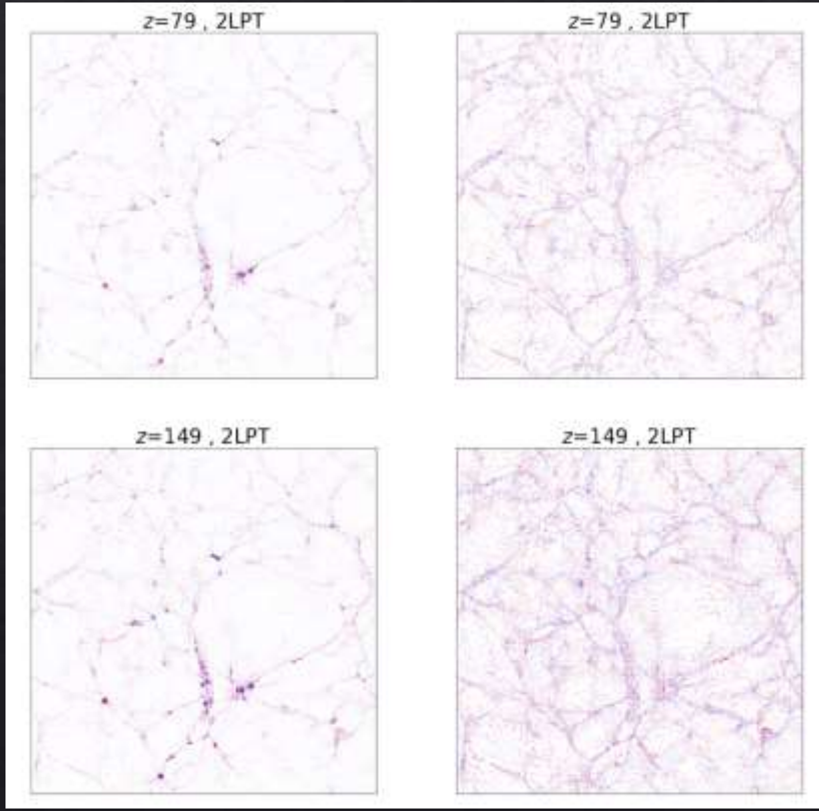


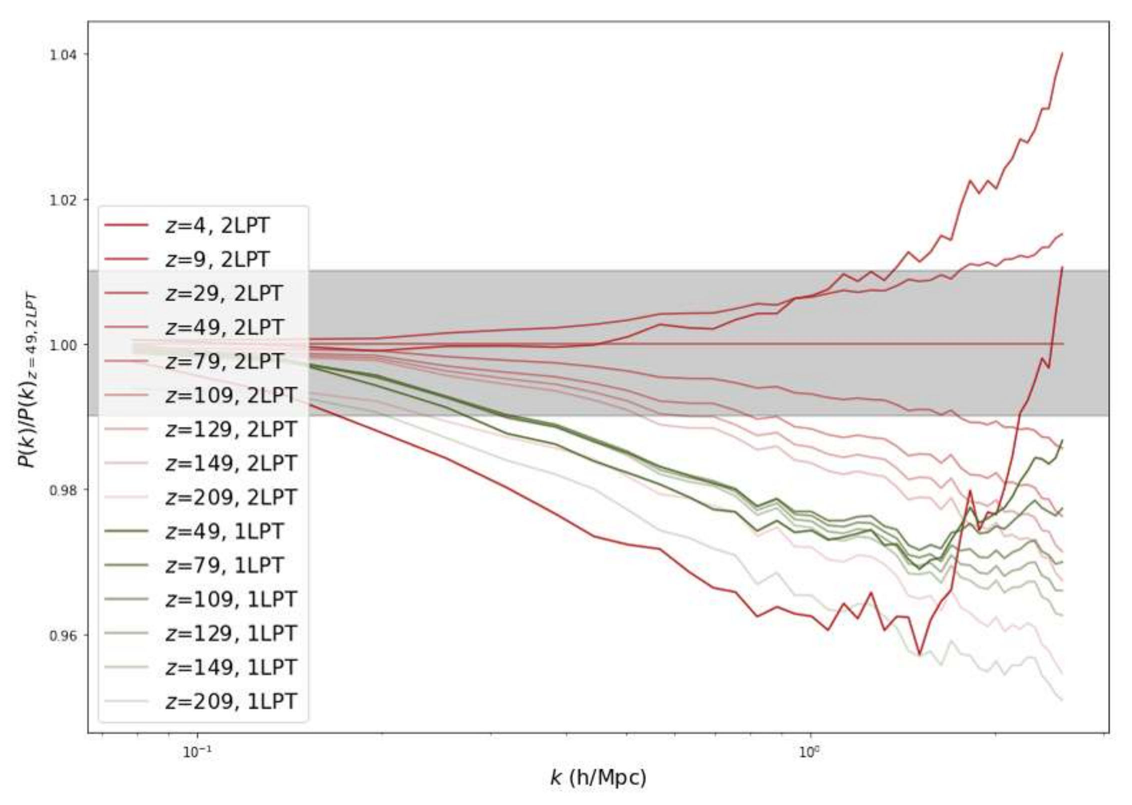


X
Y
Z
0.0
0.2
0.4
0.6
0.8
1.0

Impact of changes in the starting redshift

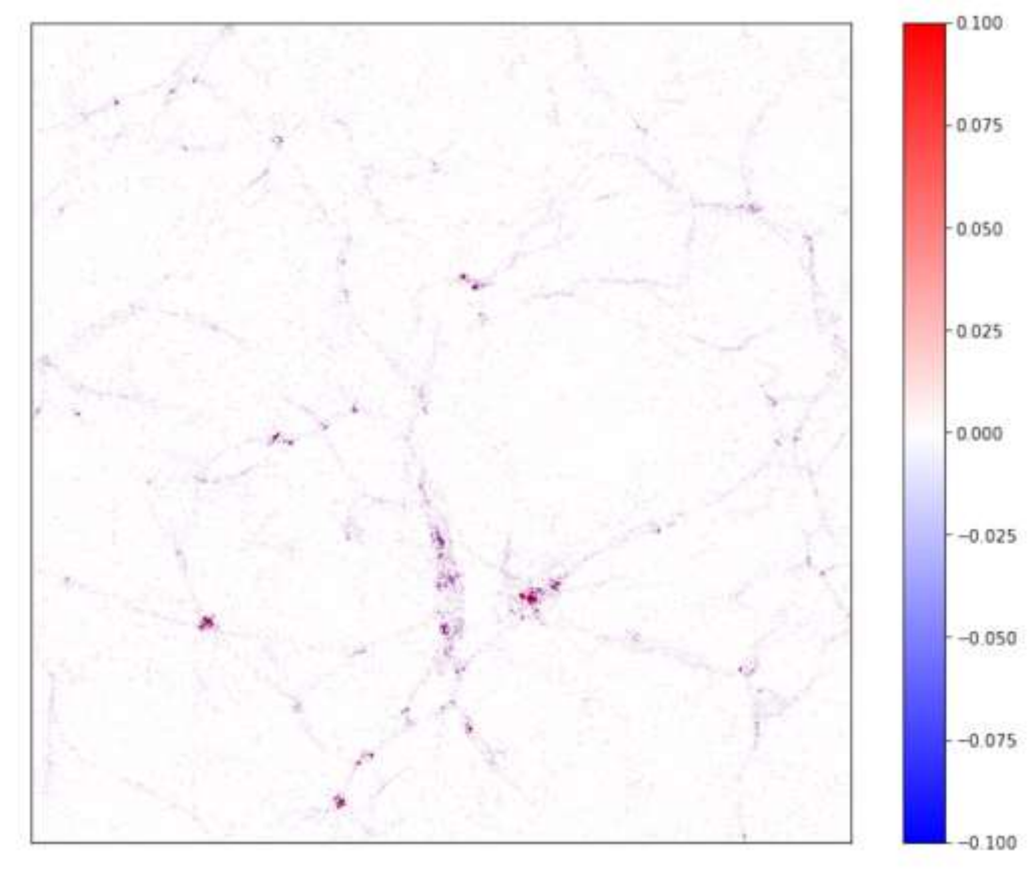
- 1LPT solutions quickly deviate from 2LPT solutions and are more sensitive to errors for later initial times
- 2LPT solutions are consistent up to around the initial redshift $z = 30$

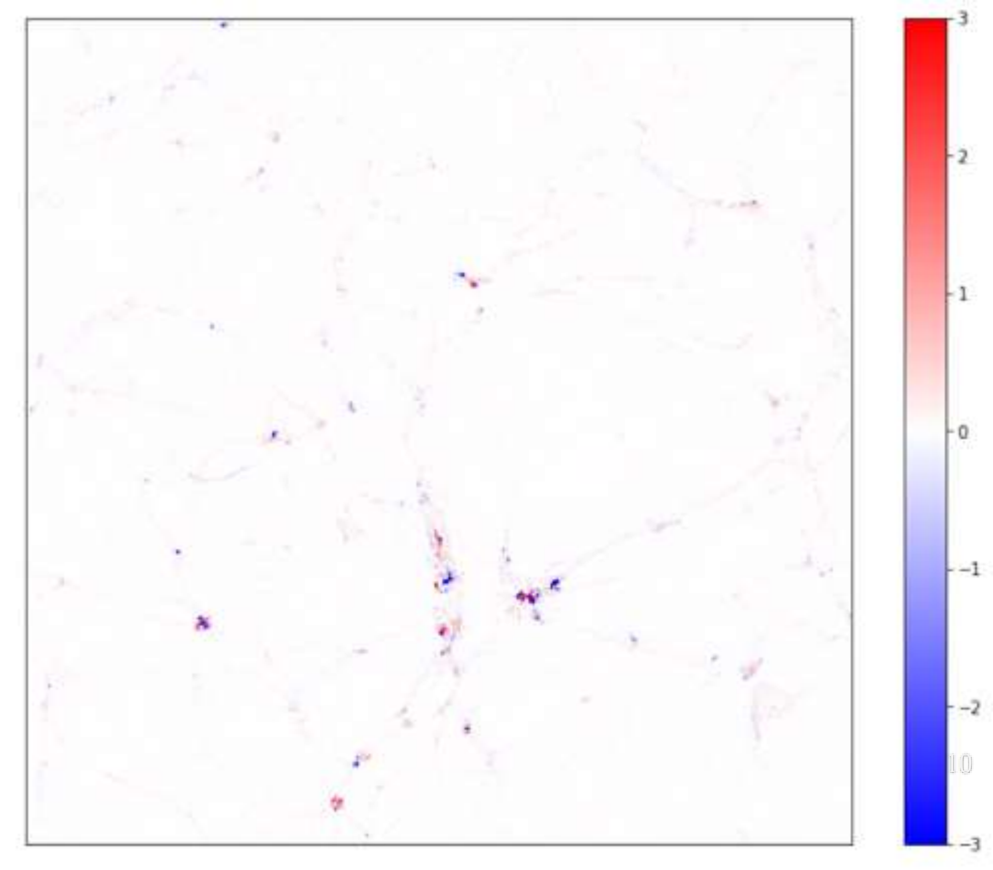




Impact of changes in the mass assignment scheme

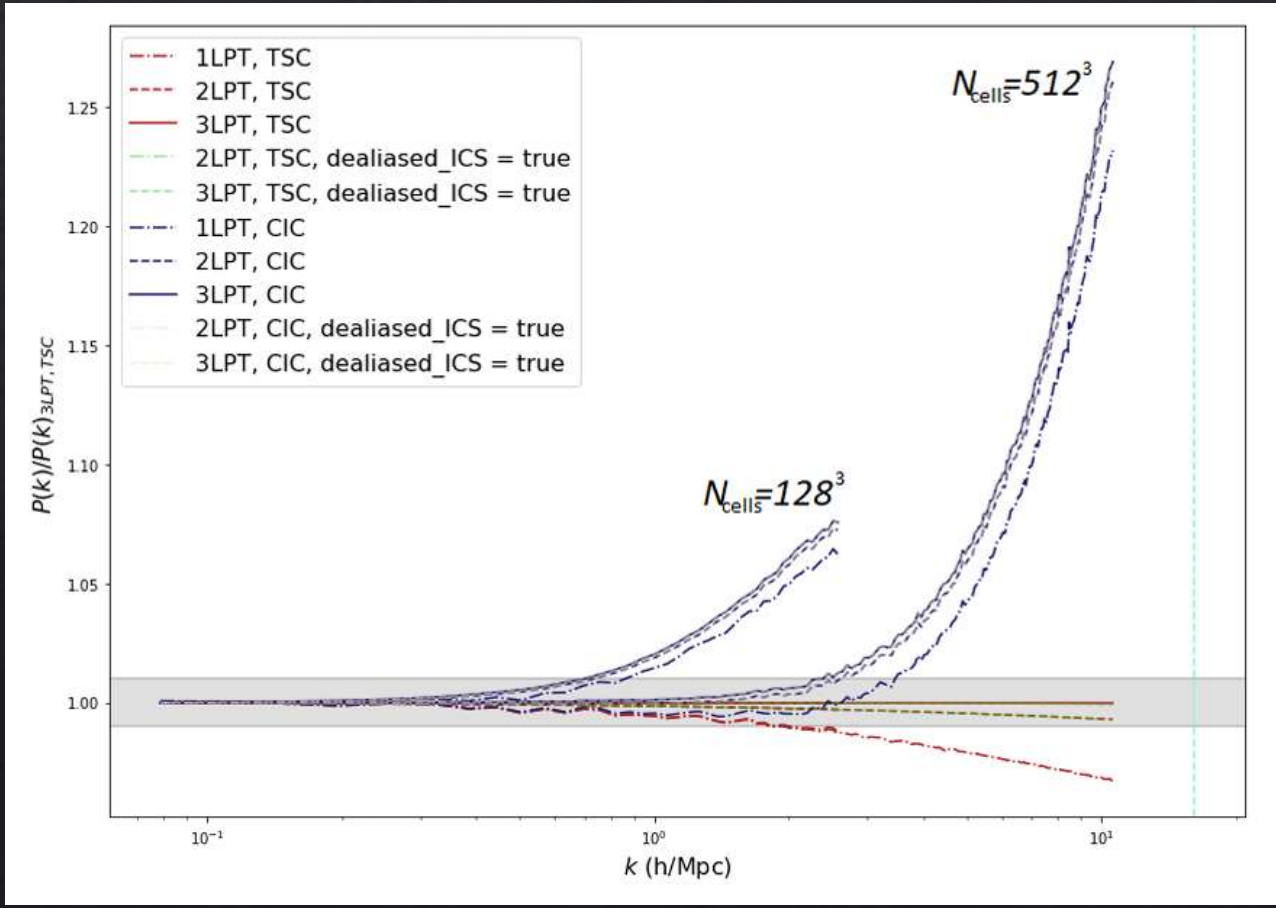
- CIC and TSC shemes





Impact of changes in the resolutins

- increasing the resolution improves the consistency of results between different mass assignment schemes



- a smaller number of particles relative to grid points allows for obtaining results for wave numbers up to the particle Nyquist wave number, beyond which results can no longer be trusted

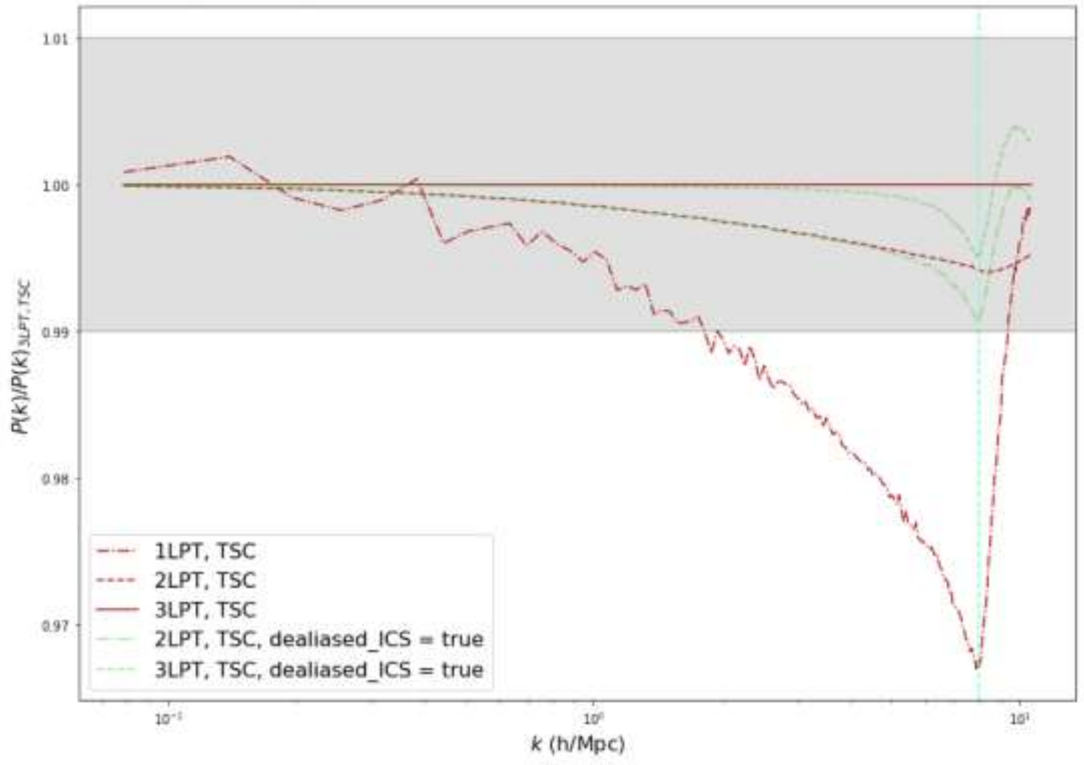


FIG. 10. matter power spectra with a resolution of 256^3 particles and 512^3 grid points, starting from $z = 49$ and ending at $z = 24$.

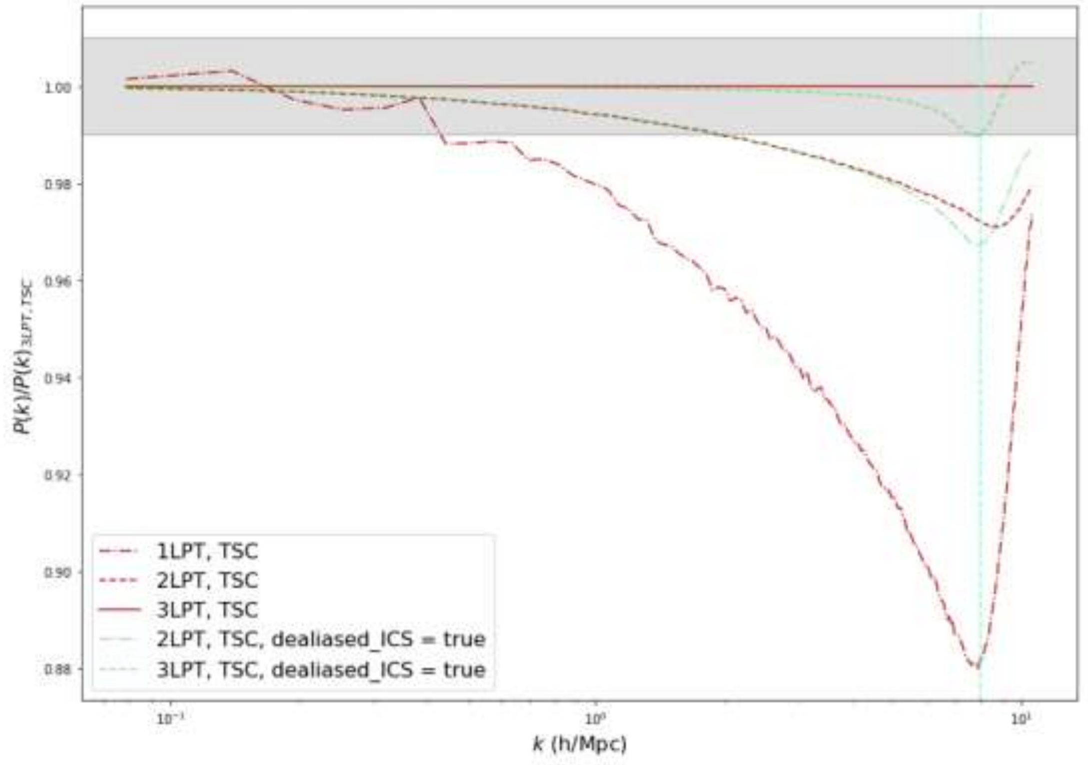


FIG. 11. Figure 11. matter power spectra with a resolution of 256^3 particles and 512^3 grid points, starting from $z = 24$ and ending at $z = 11.5$.

Conclusions

- the choice of the initial time should be such that we minimize the combined effects of errors arising from truncation and particle discretization
- using a higher order of LPT allows for this moment to be as late as possible
- although different schemes do not have a significant impact on the density calculation at a given moment, they significantly affect the final results of the simulation
- increasing the resolution improves the simulation results, and to obtain values in the matter power spectrum up to the Nyquist wave number, the number of particles should be smaller than the number of grid points

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Thank you for your attention