Generalizacija Yang-Millsove teorije

Više neabelove baždarne teorije

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Sadržaj

Uvod u Yang-Millsove teorije

[1] 1954. C. N. Yang & R. L. Mills

PHYSICAL REVIEW VOLUME 96, NUMBER 1 OCTOBER 1, 1954

Conservation of Isotopic Spin and Isotopic Gauge Invariance*

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(Received June 28, 1954)

It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistant with the concept of localized fields. The possibility is explored of having invariance under local isotopic spin rotations. This leads to formulating a principle of isotopic gauge invariance and the existence of a **b** field which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge. The **b** field satisfies nonlinear differential equations. The quanta of the **b** field are particles with spin unity, isotopic spin unity, and electric charge $\pm e$ or zero.

INTRODUCTION

THE conservation of isotopic spin is a much discussed concept in recent years. Historically an isotopic spin parameter was first introduced by Heisenberg[1] in 1932 to describe the two charge states (namely neutron and proton) of a nucleon. The idea that the neutron and proton correspond to two states of the same particle was suggested at that time by the fact that their masses are nearly equal, and that the light stable even nuclei contain equal numbers of them. Then in 1937 Breit, Condon, and Present pointed out the approximate equality of $p-p$ and $n-p$ interactions in the ^1S state.[2] It seemed natural to assume that this equality holds also in the other states available to both the $n-p$ and $p-p$ systems. Under such an assumption one arrives at the concept of a total isotopic spin[3] which is conserved in nucleon-nucleon interactions. Experi-

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† On leave of absence from the Institute for Advanced Study, Princeton, New Jersey.

[1] W. Heisenberg, Z. Physik **77**, 1 (1932).

[2] Breit, Condon, and Present, Phys. Rev. **50**, 825 (1936). J. Schwinger pointed out that the small difference may be attributed to magnetic interactions [Phys. Rev. **78**, 135 (1950)].

[3] The total isotopic spin **T** was first introduced by E. Wigner, Phys. Rev. **51**, 106 (1937); B. Cassen and E. U. Condon, Phys. Rev. **50**, 846 (1936).

Kalb-Ramondovo polje

[2] 1974. M. Kalb & P. Ramond

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Classical direct interstring action*

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We generalize the classical action-at-a-distance theory between point particles to include one-dimensionally extended objects (strings) in space-time. We build parametrization-invariant couplings which lead to equations of motion for strings in each others' influence. The direct coupling of the area elements of the world sheets of the strings is considered in detail, from which we define an antisymmetric adjunct field. We find that, for a given interaction, the nature of the forces depends on the type of strings involved, that is, open- vs closed-ended. Our coupling can be understood in terms of states appearing in the Veneziano and Shapiro-Virasoro models in 26 dimensions. However, we find an additional massive pseudovector field which arises from the interaction between the "Reggeon" and "Pomeron" sectors of this dual model.

I. INTRODUCTION

The dual resonance models,[1] whose aim is a self-contained description of the strong interactions, have of late been understood in terms of a strikingly simple and beautiful picture. On the

classical description. A difficulty in overcoming these defects is that the strings have so far been described in terms of their world sheets rather than by the fields associated with them. One may hope therefore that the development of a more powerful formalism might alleviate and perhaps

Standardna Yang-Millsova teorija

Baždarna polja Yang-Millsove teorije kao Ehresmannove koneksije na glavnom G-svežnju i tenzori jačine polja kao zakrivljenost koneksije.

Glavni G-svežanj

- Liejeva grupa G
- glatka surjekcija između glatkih mnogostrukosti, $\pi : P \to M$
- slobodna desna G-akcija na P, $\triangleleft : P \times G \to P$
- vlakna $G_p \equiv \pi^{-1}(\{p\}) \cong G$ za $p \in P$

Lokalna trivijalnost

Lokalne trivijalizacije

Neka je $\{U_i\}_{i\in I}$ otvoreni pokrivač od M. Ako postoje difeomorfizmi

$$\{\phi_i : U_i \times G \to \pi^{-1}(U_i)\}_{i\in I}$$

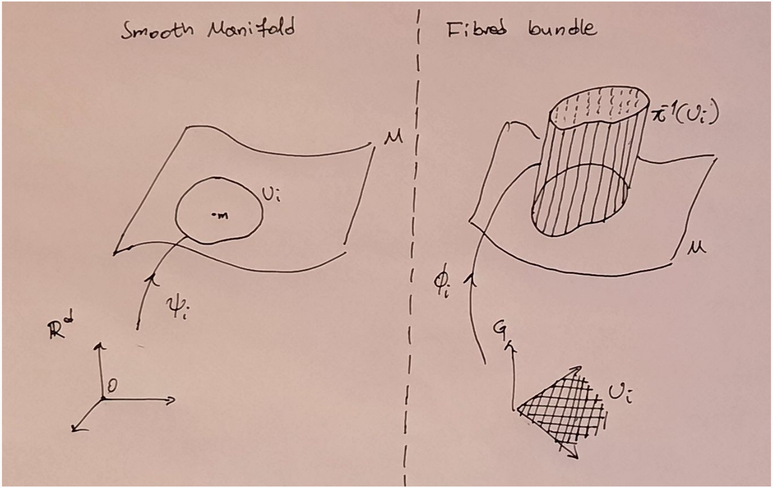
t.d.

$$(\pi \circ \phi_i)(m, g) = m$$

za $m \in M$ i $g \in G$, kaže se da je P lokalno trivijalan.

Posebno, za takve difeomorfizme se kaže da su lokalne trivijalizacije od P.

Lokalna trivijalnost



Funkcije prijelaza

Neka je $m \in M$. Kako su ϕ_i i ϕ_j difeomorfizmi, postoje $g_i, g_j \in G$ t.d. $\phi_i(m, g_i) = \phi_j(m, g_j)$.

Funkcije prijelaza

Za glatke funkcije $t_{ij} : U_i \cap U_j \to G$ t.d.

$$\phi_j(m, g_j) = \phi_i(m, g_i t_{ij}(m))$$

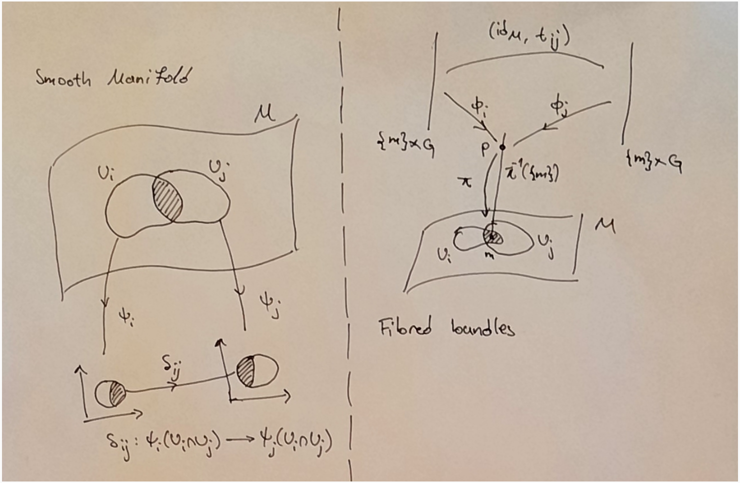
se kaže da su fukcije prijelaza.

Također se zahtijevaju sljedeće relacije:

$$\begin{aligned} t_{ii}(p) &= e \quad && (p \in U_i), \\ t_{ij}(p) &= (t_{ji}(p))^{-1} \quad && (p \in U_i \cap U_j), \\ t_{ij}(p) t_{jk}(p) &= t_{ik}(p) \quad && (p \in U_i \cap U_j \cap U_k), \end{aligned}$$

koje daju relaciju ekvivalencija između lokalnih trivijalizacija.

Funkcije prijelaza



Diferencijabilnost

G je također glatka mnogostrukost (sa posebnim tangentnim prostorom $\mathfrak{g}$). We also define a special Lie algebra isomorphism using the adjoint map.

Ehresmannova koneksija

Diferencijalna 1-forma sa vrijednostima u $\mathfrak{g}$, $\omega \in T^*P \otimes \mathfrak{g}$ je Ehresmannova koneksija ako

$$\omega(x^{\#}) = x, \text{ za sve } x \in \mathfrak{g}$$
$$(\triangleleft g)_* \omega = \mathrm{Ad}_{g^{-1}} \omega$$

Definira se vanjski produkt i Hodgeov dual kao:

$$(\omega \otimes x) \wedge (\eta \otimes y) = (\omega \wedge \eta) \otimes [x, y] \quad * (\omega \otimes x) = (*\omega) \otimes x$$

Diferencijabilnost

Za dane lokalne prereze $\{\sigma_i : U_i \to P\}_{i \in I}$ postoji korespodencija između Ehresmannove koneksije i diferencijalnih 1-formi sa vrijednostima u $\mathfrak{g}$ na T^*M.

$$A_i = \sigma_i^* \omega$$

$$\omega_i = g_i^{-1} \pi^* A_i g_i + g_i^{-1} \mathrm{d} g_i$$

gdje su g_i t.d. $\phi_i^{-1}(\sigma_i(m) \triangleleft g_i) = (m, g_i)$.

$$A_j = t_{ij}^{-1} A_i t_{ij} + t_{ij}^{-1} \mathrm{d} t_{ij} \text{ (baždarna transformacija)}$$

Paralelan transport vektora dan:

$$\mathcal{P}\exp\left(\int_\gamma A\right)$$

Standardna Yang-Millsova teorija

U slučaju $\mathbb{R}^4$, s $A = A_{a\mu}\mathrm{d}x^\mu \otimes T^a$, gdje su T^a generatori $\mathfrak{su}(2)$ algebre, $[T^a, T^b] = \epsilon^{abc}T^c$ dobije se standardna YM teorija:

$$F = \mathrm{d}A + A \wedge A = \frac{1}{2}F_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu, \quad \text{gdje}$$
$$F_{\mu\nu} = F_{a\mu\nu} \otimes T^a = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$
$$F_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + \epsilon_{abc}A_{b\mu}A_{c\nu}.$$

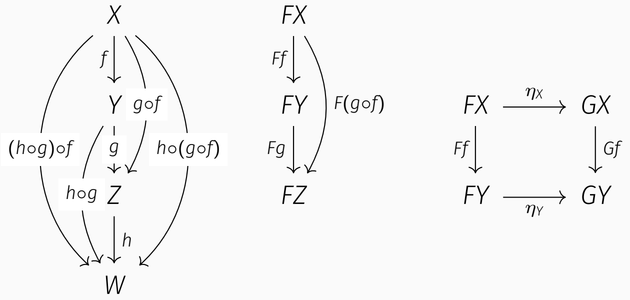
za tenzor jačine polje te:

$$S_{YM} = -\frac{1}{4}\int_{\mathbb{R}^4} \mathrm{d}^4x\, \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}),$$
$$\mathcal{D} * F = 0 \iff \mathcal{D}_\mu F^{\mu\nu} = 0$$

za akciju i jednadžbe gibanja.

Kategorije

Kategorije



- Yoneda
- Reprezentabiilnost
- Limiti

Kategorifikacija Yang-Millsove teorije

Homotopije za mnogostrukosti: grupoid puteva

Put

Neka su $x, y \in M$. Put između x i y je glatko preslikavanje $\gamma : [0,1] \to M$ za koje postoji $\epsilon > 0$ t.d.:

$$\begin{aligned} x &= \gamma(t) \quad \text{za} \quad t \in [0, \epsilon], \\ y &= \gamma(t) \quad \text{za} \quad t \in [1-\epsilon, 1] \end{aligned}$$

Produkt puteva

Za dva puta u M, γ_1 od x do y i γ_2 od y do z, definira se put $\gamma_1\gamma_2$ od x do z:

$$\gamma_1\gamma_2(t) = \begin{cases} \gamma_1(2t) & \text{za } t \in [0, 1/2] \\ \gamma_2(2t-1) & \text{za } t \in (1/2, 1] \end{cases}$$

Homotopije za mnogostrukosti: grupoid puteva

Homotopija

Neka su $x, y \in M$ te γ_1, γ_2 putevi od x do y. Glatko preslikavanje $H : [0,1]^2 \to M$ je homotopija ako postoji $\epsilon > 0$ t.d.:

$$\begin{aligned} x = H(s,t) \quad & \text{za} \quad t \in [0,\epsilon], s \in [0,1] \\ y = H(s,t) \quad & \text{za} \quad t \in [1-\epsilon,1], s \in [0,1] \\ \gamma_1(t) = H(s,t) \quad & \text{za} \quad s \in [0,\epsilon], t \in [0,1] \\ \gamma_2(t) = H(s,t) \quad & \text{za} \quad s \in [1-\epsilon,1], t \in [0,1] \end{aligned}$$

and

$$\mathrm{rang}[\mathrm{d}H(s,t)] \leq 1 \quad \text{for } (s,t) \in [0,1]^2.$$

Homotopije definiraju relaciju ekvivalencije na prostoru svih puteva.

Homotopije za mnogostrukosti: grupoid puteva

Grupoid puteva

Definira se kategorija, $\mathcal{P}_1(M)$ tzv. grupoid puteva na M gdje,

- $\mathrm{obj}(\mathcal{P}_1(M)) = M$,
- $Hom(x, y) = \{\text{klase ekvivalencija puteva od x do y}\}$.

Lokalna trivijalnost

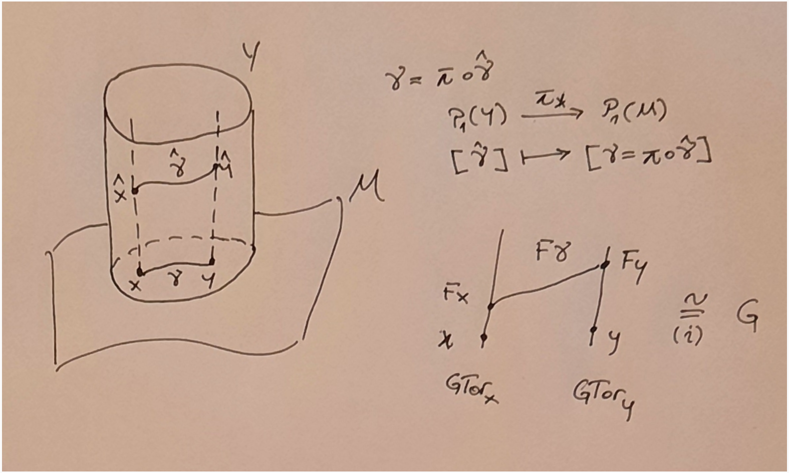
Neka je $\pi : Y \to M$ surjektivna submerzija između glatkih mnogostrukosti, te $F : \mathcal{P}_1(M) \to GTor$ funktor.

Lokalna trivijalizacija

Dijagrami oblika:

$$\begin{array}{ccc} \mathcal{P}_1(Y) & \xrightarrow{\pi_*} & \mathcal{P}_1(M) \\ {\scriptstyle triv}\downarrow & \overset{t}{\swarrow\!\!\!\!\!\swarrow} & \downarrow{\scriptstyle F} \\ Gr & \xrightarrow[i]{} & GTor \end{array}$$

gdje je *triv* funktor, t prirodna ekvivalencija, i ekvivalencija kategorija, zovu se lokalne trivijalizacije funktora F.


Y
M
γ = π ∘ γ̂
P1(Y) —π*→ P1(M)
[γ̂] ↦ [γ = π∘γ̂]
x̂
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ŷ
x
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y
Fγ
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x
y
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(i)
G
GTorx
GTory

Lokalna trivijalnost

Kategorija lokalnih trivijalizacija $Triv_1(i, \pi)$

Za fiksne i i π definira se kategorija $Triv_1(i, \pi)$ kao

- $obj(Triv_1(i, \pi)) = \{$lokalne trivijalizacije funktora$\}$
- morfizmi su dani prirodnim transformacijama među raznim $\mathcal{P}_1(M) \to GTor$ funktorima.

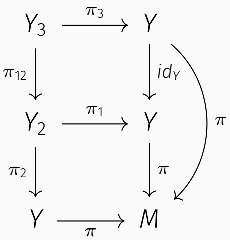
Funkcije prijelaza

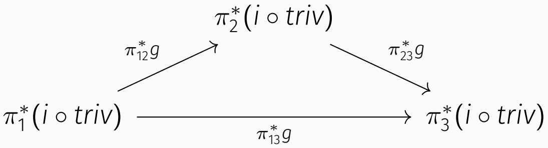
$$\begin{array}{ccc} Y_2 & \xrightarrow{\pi_1} & Y \\ {\scriptstyle \pi_2}\downarrow & & \downarrow{\scriptstyle \pi} \\ Y & \xrightarrow[\pi]{} & M \end{array} \qquad \begin{array}{c} \mathcal{P}_1(Y_2) \\ {\scriptstyle \pi_{1*}}\downarrow \; \downarrow{\scriptstyle \pi_{2*}} \\ \mathcal{P}_1(Y) \\ \downarrow{\scriptstyle i \circ triv \cong \pi_* F} \\ GTor \end{array}$$

$$\pi_* F \circ \pi_{2*} = F \circ \pi \circ \pi_2 = F \circ \pi \circ \pi_1$$

$g \equiv \pi_2^* t \circ \pi_1^* t^{-1}$ je prirodna ekvivalencija

Funkcije prijelaza





Kategorija $\mathrm{Des}_1(i)$

Objekti su komutativni trokuti dani prirodnim ekvivalencijama g, morfizmi su prizme kojima su baze dane trokutima.

Ekvivalencija

[3]Postoji sljedeća ekvivalencija među kategorijama:

$$Triv_1(i,\pi) \cong Des_1(i) \cong Fun_\infty(\mathcal{P}_1(Y), \mathcal{B}G) \cong \Omega_1(M, \mathfrak{g})$$

Više kategorije

Svežnjevi u višim kategorijama

[4][5][6]

$$\begin{array}{ccc} \mathcal{P}_n(Y) & \xrightarrow{\pi_*} & \mathcal{P}_n(M) \\ {\scriptstyle triv}\downarrow & & \downarrow{\scriptstyle F} \\ Gr & \xrightarrow[i]{} & GTor \end{array}$$

Reference

C. N. Yang and R. L. Mills.
Conservation of isotopic spin and isotopic gauge invariance.
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