

8. NEPOSREDNO INTEGRIRANJE I METODA SUPSTITUCIJE

- $\int kf(x)dx = k \int f(x)dx$
- $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$

$$\int x^a dx = \frac{x^{a+1}}{a+1} + c \quad (a \neq -1)$$

$$\int \frac{dx}{x} = \ln|x| + c$$

$$\int e^x dx = e^x + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c \quad (a > 0, a \neq 1)$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + c$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + c \quad (a > 0)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left(x + \sqrt{a^2 + x^2} \right) + c \quad (a > 0)$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right| + c \quad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + c \quad (a > 0)$$

8.1. Izračunajte neodređene integrale:

$$(a) \int \left(1 - \frac{1}{x^2}\right) \sqrt{x\sqrt{x}} \, dx$$

$$(b) \int \frac{x^2 + 5x - 1}{\sqrt{x}} \, dx$$

$$(c) \int \frac{x^2}{x^2 + 1} \, dx$$

$$(d) \int \frac{2^x + 5^x}{10^x} \, dx$$

$$(e) \int \frac{2^{x+1} - 5^{x-1}}{10^x} \, dx$$

$$(f) \int \frac{1}{\sin^2 x \cos^2 x} \, dx$$

$$(g) \int \operatorname{tg}^2 x \, dx$$

8.2. Izračunajte određene integrale:

(a) $\int_0^1 (3x^2 - x + 2) dx$

(b) $\int_{-\sqrt{5}}^{\sqrt{5}} \frac{dx}{x^2 + 5}$

(c) $\int_{\sqrt{2}}^2 \frac{dx}{\sqrt{4 - x^2}}$

8.3. Izračunajte neodređene integrale:

(a) $\int \sin 3x dx$

(b) $\int e^{3 \cos x} \sin x dx$

(c) $\int \frac{(\ln x)^4}{x} dx$

(d) $\int x \sqrt{1 + x^2} dx$

(e) $\int \frac{dx}{x - 7}$

(f) $\int \frac{dx}{1 + e^x}$

8.4. Izračunajte neodređene integrale:

(a) $\int \frac{dx}{x\sqrt{2x-9}}$

(b) $\int \frac{x^2}{\sqrt[3]{5+x^3}} dx$

(c) $\int \frac{\sin \sqrt[3]{x}}{\sqrt[3]{x^2}} dx$

(d) $\int \frac{\sqrt{x} + \ln x}{x} dx$

(e) $\int \frac{e^{2x}}{1-3e^{2x}} dx$

(f) $\int \frac{\cos x}{1+2\sin x} dx$

(g) $\int \frac{\cos 2x}{\sin x \cos x} dx$

(h) $\int \sin^2 x dx$

(i) $\int \cos^2 x dx$

(j) $\int \cos^5 x \sqrt{\sin x} dx$

8.5. Izračunajte neodređeni integral

$$\int \frac{e^{\operatorname{arctg} x} + x \ln(1 + x^2) + 1}{1 + x^2} dx.$$

8.6. Izračunajte određene integrale:

(a) $\int_0^{\frac{\pi}{3}} \frac{\sin^3 x}{\cos^4 x} dx$

(b) $\int_{-1}^2 \frac{dx}{(3 + 2x)^2}$

(c) $\int_1^e \frac{dx}{x\sqrt{1 + \ln x}}$

(d) $\int_0^4 \frac{dx}{1 + \sqrt{x}}$

(e) $\int_0^{\ln 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx$

(f) $\int_0^6 (x - 3)e^{x^2 - 6x} dx$

- 8.1. (a) $\frac{4(x^2 + 7)}{7\sqrt[4]{x}} + c$
- (b) $\frac{2\sqrt{x}}{15}(3x^2 + 25x - 15) + c$
- (c) $x - \operatorname{arctg} c + c$
- (d) $-\frac{5^{-x}}{\ln 5} - \frac{2^{-x}}{\ln 2} + c$
- (e) $-2\frac{5^{-x}}{\ln 5} - \frac{2^{-x}}{5\ln 2} + c$
- (f) $\operatorname{tg} x - \operatorname{ctg} x + c$
- (g) $-x + \operatorname{tg} x + c$
- 8.2. (a) $\frac{5}{2}$
- (b) $\frac{\pi}{2\sqrt{5}}$
- (c) $\frac{\pi}{4}$

8.3. (a) $-\frac{\cos(3x)}{3} + c$

(b) $-\frac{e^{3 \cos x}}{3} + c$

(c) $\frac{(\ln x)^5}{5} + c$

(d) $\frac{1}{3}(1+x^2)\sqrt{1+x^2} + c$

(e) $\ln|x-7| + c$

(f) $x - \ln(1 + e^x) + c$

$$8.4. \quad (a) \quad \frac{2}{3} \operatorname{arctg} \left(\frac{1}{3} \sqrt{2x-9} \right) + c$$

$$(b) \quad \frac{1}{2} \sqrt[3]{(5+x^3)^2} + c$$

$$(c) \quad -3 \cos \sqrt[3]{x} + c$$

$$(d) \quad 2\sqrt{x} + \frac{\ln^2 x}{2} + c$$

$$(e) \quad -\frac{1}{6} \ln |1 - 3e^{2x}| + c$$

$$(f) \quad \frac{1}{2} \ln |1 + 2 \sin x| + c$$

$$(g) \quad \ln |\sin 2x| + c$$

$$(h) \quad \frac{x}{2} - \frac{1}{4} \sin 2x + c$$

$$(i) \quad \frac{1}{2} (x + \cos x \sin x) + c$$

$$(j) \quad \frac{2}{3} (\sin x)^{3/2} - \frac{4}{7} (\sin x)^{7/2} - \frac{2}{11} (\sin x)^{11/2} + c$$

$$8.5. \quad e^{\operatorname{arctg} x} + \frac{1}{4} \ln^2(1+x^2) + \operatorname{arctg} x + c$$

- 8.6. (a) $\frac{4}{3}$
(b) $\frac{3}{7}$
(c) $2\sqrt{2} - 2$
(d) $4 - 2\ln 3$
(e) $4 - \pi$