14. NIZOVI I NEPRAVI INTEGRAL

14.1. Izračunajte limese sljedećih nizova:

(a) $a_n = \dfrac{2n^3 - 1}{2 - n^3}$

(b) $a_n = \dfrac{2n^3 + 3}{4n - 1}$

(c) $a_n = \dfrac{(3n + 1)(2n - 3)(5 - 2n)}{(4 - 3n)(7 + n)(2n - 1)}$

(d) $a_n = \dfrac{3}{2 \operatorname{arctg} n}$

(e) $a_n = \dfrac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$

(f) $a_n = \dfrac{(-3)^n + 4^n}{(-3)^{n+1} + 4^{n+1}}$

(g) $a_n = \dfrac{n^2(n^2 + 1)}{2^n(n^2 - 1)}$

(h) $a_n = \dfrac{(n + 1)(3^n + 1)}{2 \cdot 3^n + 1}$

14.2. Izračunajte limese sljedećih nizova:

(a) $a_n = \sqrt{n^2+n} - n$

(b) $a_n = \dfrac{\sqrt{n^4+n^3+1} - n^2}{2n+1}$

(c) $a_n = \left(1 - \dfrac{1}{3n}\right)^n$

(d) $a_n = \left(\dfrac{n+1}{n-1}\right)^n$

(e) $a_n = \left(\dfrac{2n+3}{2n+1}\right)^{(n+1)}$

(f) $a_n = n[\ln(n+3) - \ln n]$

(g) $a_n = \dfrac{2^n}{\ln\left(1+\frac{1}{2n}\right)^{n+1}}$

(h) $a_n = n\sqrt[n]{\sin\dfrac{2}{n}}$

14.3. Izračunajte limese sljedećih nizova:

(a) $a_n = \dfrac{\sqrt[3]{n^2}\sin n}{n+1}$

(b) $a_n = \dfrac{n\sin(n!)}{n^2+1}$

14.4. Izračunajte limese sljedećih nizova zadanih rekurzivno ako postoje:

(a) $a_1 = 0, a_{n+1} = \dfrac{a_n^2+1}{2}, n \in \mathbb{N}$

(b) $a_1 = 3, a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{3}{a_n}\right), n \in \mathbb{N}$

(c) $a_1 = \sqrt{2}, a_{n+1} = \sqrt{2+a_n}, n \in \mathbb{N}$

(d) $a_1 = 3, a_{n+1} = \dfrac{3}{n+1}a_n, n \in \mathbb{N}$

14.5. Izračunajte sva gomilišta sljedećih nizova:

(a) $a_n = \dfrac{3n^2+2n}{n^2-1} \cdot \dfrac{1+(-1)^n}{2} + \dfrac{1-(-1)^n}{n}$

(b) $a_n = \left(1+\dfrac{1}{n}\right)\cos n\pi$

14.6. Izračunajte limese sljedećih nizova:

(a) $a_n = \frac{1}{n^3} \sum_{k=1}^{n} k(k+1)$

(b) $a_n = \frac{1}{n+1} \sum_{k=1}^{n} (2k-1) - \frac{2n+1}{2}$

(c) $a_n = \frac{1}{n^3} \sum_{k=1}^{n} \sum_{i=1}^{k} i$

(d) $a_n = 1 + 2 + 4 + 8 + \cdots + 2^{n-1}$

(e) $a_n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$

(f) $a_n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots + \frac{(-1)^n}{2^n}$

(g) $a_n = \prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right)$

(h) $a_n = \prod_{k=2}^{n} \frac{k^3 - 1}{k^3 + 1}$

(i) $a_n = \prod_{k=2}^{n} \frac{k^2 + k - 2}{k(k+1)}$

14.7. Ukoliko konvergiraju, izračunajte sljedeće neprave integrale:

(a) $\int_e^{\infty} \frac{dx}{x \ln^3 x}$

(b) $\int_0^{\infty} xe^{-x^2}\, dx$

(c) $\int_{-\infty}^{0} \frac{1}{\sqrt{3-x}} dx$

(d) $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

(e) $\int_{-\infty}^{\infty} \frac{dx}{x^2+4x+9}$

(f) $\int_0^{\frac{\pi}{2}} \operatorname{ctg} x \sqrt[3]{\sin x} dx$

(g) $\int_1^2 \frac{x-2}{\sqrt{x-1}} dx$

(h) $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$

(i) $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

14.8. Ispitajte konvergenciju sljedećih nepravih integrala:

(a) $\int_2^\infty \frac{\cos^2 x}{x^2}\,dx$

(b) $\int_1^\infty \frac{e^{-x}}{x}\,dx$

(c) $\int_1^\infty e^{-x^2}\,dx$

(d) $\int_1^\infty \frac{3+\sin x}{\sqrt[3]{x}}\,dx$

(e) $\int_1^\infty \frac{1+3\sin^4(2x)}{\sqrt{x}}\,dx$

(f) $\int_1^\infty \frac{x}{1+x^2\sin^2 x}\,dx$

(g) $\int_3^\infty \frac{1}{x+e^x}\,dx$

(h) $\int_3^\infty \frac{1}{x-e^x}\,dx$

Rješenja

14.1. (a) -2
(b) ∞
(c) 2
(d) $3/\pi$
(e) 3
(f) $1/4$
(g) 0
(h) ∞

14.2. (a) $1/2$
(b) $1/4$
(c) $1/\sqrt[3]{e}$
(d) e^2
(e) e
(f) 3
(g) ∞
(h) ∞

14.3. (a) 0
(b) 0

14.4. (a) 1
(b) $\sqrt{3}$
(c) 2
(d) 0

14.5. (a) $0, 3$
(b) $-1, 1$

14.6. (a) $1/3$
(b) $-3/2$
(c) $1/6$
(d) ∞
(e) 2
(f) $2/3$
(g) $1/2$
(h) $2/3$
(i) $1/3$

14.7. (a) $1/2$
(b) $1/2$
(c) divergira
(d) π
(e) $\pi/\sqrt{5}$
(f) 3
(g) $-4/3$
(h) 1
(i) π

14.8. (a) konvergira
(b) konvergira
(c) konvergira
(d) divergira
(e) divergira
(f) divergira
(g) konvergira
(h) divergira