

DIR1, zadaća 2, 13.03.2007.

KORJENI

1. odredite domene sljedećih funkcija:

a) $f(x) = \sqrt{x^2 - x + 1}$

b) $f(x) = \sqrt[4]{x^2 + 9x + 4} - 4$

c) $f(x) = \sqrt[120]{(x^2 + 5x + 6)(x + 3)}$

2. Riješite sljedeće jednačbe:

a) $\sqrt{(2x - 1)^2 - 16} = 1 - x$

b) $\sqrt{2x + 6} - \sqrt{2x + 4} = 1$

c) $11x + \sqrt{2x^2 - 11x + 9} = 7$

d) $\sqrt[2008]{\sqrt{|x^2 - 4x + 1| + 3} + 1} = 1$

3. Dokažite da, ako je f rastuća bijekcija, onda je i f^{-1} rastuća bijekcija. Naravno, analogno vrijedi i za padajuću bijekciju.

4. Riješite sljedeće nejednačbe:

a) $\sqrt[4]{x^2 - 11x - 12} \geq \sqrt[3]{2}\sqrt[4]{3}$

b) $\sqrt{x^2 - 2x + 6} \leq \sqrt{x^2 + 2x + 6}$

c) $\sqrt{1 + x + x^2} \leq \sqrt{1 - x + x^2}$

d) $\sqrt[3]{(x + 1)^2} \geq \sqrt[3]{(x - 1)^2}$

POLINOMIJALNE FUNKCIJE

5. Odredite polinom P stupnja 3 ako je poznato $P(0) = 0$, $P(1) = 3$, $P(2) = 14$, $P(3) = 39$.

6. Skicirajte grafove polinoma:

a) $P(x) = x^4 + x^3 - 2x^2$

b) $P(x) = (2x + 1)(3x + 1)(4x + 1)$

c) $P(x) = (x^2 - 1)(x - 1)$

d) $P(x) = x^3 - 6x^2 + 11x - 6$

RACIONALNE FUNKCIJE

7. Riješite sljedeće (ne)jednačbe:

a)

$$\frac{x^3 + 7x + 24}{x^2 - 5x + 6} = 4$$

b)

$$\frac{x^2 + x + 2}{x^2 + 2x + 1} = 1$$

c)

$$\frac{(x+6)(x+3)}{(x-6)(x-3)} \leq 0$$

TRIGONOMETRIJSKE FUNKCIJE

8. Skicirajte grafove funkcija:

1) $f(x) = 2 \sin x$

2) $f(x) = \sin 2x$

3) $f(x) = \sin(x/2)$

4) $f(x) = \sin(x + \pi/2)$

5) $f(x) = \sin(2x + \pi/2)$

6) $f(x) = 3 \sin(2x + \pi/2) - 1$

7) $f(x) = \frac{1}{2} \sin(\frac{1}{2}(x + 2\pi)) + 2$

8) $f(x) = |\sin x|$

1) $f(x) = \sin |x|$

9. Riješite sljedeće jednačbe:

a) $\sin x = \cos x$

b) $\frac{\sin x + 2}{2 \cos x + 1} = \frac{\cos x + 2}{2 \sin x + 1}$

c) $\cos 2x = \sqrt{2}(\cos^3 x + \sin^3 x - \sin x \cos^2 x - \sin^2 x \cos x)$

d) $\sin 2x = -\cos(x + \frac{\pi}{2})$

10. Riješite sljedeće nejednačbe:

a) $\sin x < \cos 2x$

b) $|\frac{\sin x}{\cos x}| \leq 1$

c) $\sin(2(x + \frac{\pi}{4})) \cdot \cos 4x \leq \cos 2x \cdot \cos 3x$

EKSPONENCIJALNE I LOGARITAMSKE FUNKCIJE

11. Skicirajte grafove funkcija:

a) $f(x) = 3 \cdot 2^{x-1} + 4$

b) $f(x) = 2^{2x+1} - 4$

c) $f(x) = 5 \cdot 3^{3x-4} + 2$

12. Riješite jednačbe:

a) $(\frac{1}{8})^x = 64$

b) $0,125^{2-3x} = 1/32$

- c) $2^{2(x-1)} = 9^{x+1}$
- d) $0,25 \cdot \sqrt[3]{4^{2x-1}} = 8^{-2/3}$
- e) $4^{x^2} = 64$
- f) $2^{2x+2} - 6^x - 2 \cdot 3^{2x+2} = 0$
- g) $7^{x-1} + 7^x = 8^x$
- h) $3^x + 3^{x+1} = 2^{x+2}$

13. Riješite nejednadžbe:

- a) $2^{2-\frac{2}{x}} - 2^{1-\frac{2}{x}} < 1$
- b) $8 \cdot 0,5^{x(x+1)} > 0,25^{3x/2}$

14. Izračunajte

- a) $\log_{10} \frac{1}{100} =$
- b) $\log_{\frac{1}{2}} 16 =$
- c) $\log_5 5\sqrt{5} =$
- d) $(\frac{1}{3})^{-2-\log_9 25} =$
- e) $(0,01)^{\log 6 - \frac{1}{2} \log 24} =$
- f) $25^{\log \frac{1}{5} 4 + \log_{\sqrt{5}} 4} =$

15. Riješite jednadžbe:

- a) $\log_{\frac{1}{3}} x = -2$
- b) $\log_4 x = -2$
- c) $\log_x 8 = 1,5$
- d) $\log x = \frac{1}{2} \log 36 - \log 2$
- e) $\log(x-1) + \log(x-2) = 2 \log(x-3)$
- f) $\log(3x-2) - 2 = \frac{1}{2} \log(x+2) - \log 50$
- g) $\frac{2 \log x}{\log x - 1} = \frac{2}{\log x - 1}$
- h) $\log_3(1 + \log_2(1 + \log_4(1 + \log_{\frac{1}{2}}(x)))) = 0$

16. Riješite nejednadžbe:

- a) $\log_{\frac{1}{2}}(3x-1) > 0$
- b) $\log_3 \frac{x-2}{x} < 2$
- c) $\log_x 2 \cdot \log_2(4x) > 1$
- d) $\log_{\frac{1}{2}} x - \log_{2x} \frac{1}{4} > 0$
- e) $\log x^2 - (\log x)^2 \leq 3$
- f) $\frac{\log_{\frac{1}{x}}(x^2-1)}{x^2-x+1} \leq 0$

17. Odredite prirodnu domenu sljedećih funkcija:

1)

$$f(x) = \sqrt{\log_e \frac{x-4}{x+2}} + \sqrt{4-3x-x^2}$$

2)

$$f(x) = \ln(\sin(x-3)) + \sqrt{16-x^2} + \sqrt{2^x+1}$$

3)

$$f(x) = \frac{\ln(x+3)}{\sqrt{|x|-x}}$$

4)

$$f(x) = \ln(x-|x|)$$

5)

$$f(x) = \log(\sin(\frac{3x-5}{2}\pi))$$

6)

$$f(x) = \ln(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}+x})$$

7)

$$f(x) = \sqrt{\frac{(x-3)^6(3x-7)^5}{(2x+1)^{17}(2x^2-12x+18)^3}}$$

8)

$$f(x) = \log_2 \frac{x^2-5x+6}{5x^2+9x-2}$$

9)

$$f(x) = \log(\operatorname{tg} x)$$

10)

$$f(x) = \operatorname{tg}(\log x)$$

11)

$$f(x) = \ln \cos \frac{x-1}{3}$$

12)

$$f(x) \log_{x+2} \frac{x-3}{-x+1}$$

13)

$$f(x) = \sin \log_{3x} \frac{|x|}{x^2-5x}$$

14)

$$f(x) = \sqrt{\log_{x+1}^2(x+2)-1}$$