

DIR1, zadaća 4, 05.04.2007.

1. Izračunajte limese:

a)

$$\lim_{x \rightarrow 0} \frac{1 - e^{-x}}{\sin x}$$

b)

$$\lim_{x \rightarrow 0} \frac{\ln \cos x}{x^2}$$

c)

$$\lim_{x \rightarrow 0} \frac{8^x - 7^x}{6^x - 5^x}$$

d)

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} m x}{\sin n x}$$

e)

$$\lim_{x \rightarrow +\infty} \frac{2^x + 3}{2^x - 3}$$

f)

$$\lim_{x \rightarrow 0} \frac{5^x - 4^x}{x^2 + x}$$

g)

$$\lim_{x \rightarrow 1} \frac{x^x - 1}{x \ln x}$$

h)

$$\lim_{x \rightarrow +\infty} \left(\frac{x+a}{x+b} \right)^{x+c}$$

i)

$$\lim_{x \rightarrow +\infty} x(\sqrt[x]{a} - 1)$$

j)

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{\pi - 4x}$$

2. a)

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$$

b)

$$\lim_{x \rightarrow 1} \frac{\sin x - 1}{x^2 - 1}$$

c)

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 14x}$$

d)

$$\lim_{x \rightarrow +\infty} \frac{\sin x}{x}$$

e)

$$\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$$

f)

$$\lim_{x \rightarrow 0} x \sin \frac{1}{x}$$

g)

$$\lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\pi - x}$$

h)

$$\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2}$$

g)

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x}{x}$$

h)

$$\lim_{x \rightarrow 0} \frac{\operatorname{arctg} 2x}{\sin 3x}$$

i)

$$\lim_{x \rightarrow 1} \frac{\cos \frac{\pi x}{2}}{1 - \sqrt{x}}$$

3. a)

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{\sin x - x}}$$

b)

$$\lim_{x \rightarrow +\infty} \left(\frac{\sin x}{x} \right)^{\frac{1}{\sin x - x}}$$

c)

$$\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}}$$

d)

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\ln(1 + x)}$$

e)

$$\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\ln(1 + x^2)}$$

f)

$$\lim_{x \rightarrow -\infty} \frac{\ln(1 + e^x)}{\sqrt{e^x}}$$

g)

$$\lim_{x \rightarrow 0} \frac{e^{5x} - e^{2x}}{x}$$

h)

$$\lim_{x \rightarrow \frac{\pi}{2}} (1 + \operatorname{ctg} x)^{\operatorname{tg} x}$$

i)

$$\lim_{x \rightarrow 0} \frac{1 - e^{-x}}{\sin x}$$

4. Što se dešava s korijenima kvadratne jednadžbe

$$ax^2 + bx + c = 0$$

ako koeficijent a teži k nuli, a koeficijenti b i c su konstanti, pri čemu je b različit od 0? Zašto?

5. Izračunajte konstante b i k iz jednadžbe

$$\lim_{x \rightarrow +\infty} \left(kx + b - \frac{x^3 + 1}{x^2 + 1} \right) = 0$$

NEKA RJEŠENJA

1.
 - a) 1
 - b) $-1/2$
 - c) $\frac{\ln 8/7}{\ln 6/5}$
 - d) m/n
 - e) 1
 - f) $\ln 5/4$
 - g) 1
 - d) e^{a-b}
 - e) $\ln a$
 - f) $-1/2\sqrt{2}$