Alps-Adria Seminar

10th meeting

Zagreb, November 22, 2025

Department of Mathematics
Faculty od Science
University of Zagreb
Bijenička cesta 30
10000 Zagreb, Croatia

Schedule

All talks will be in Lecture Room A002

10:00–10:25 Mateo Tomašević: *Classification of Jordan multiplicative maps on matrix algebras*

10:30–10:55 Luka Boc Thaler: *On the exponential tower functions*

11:00–11:25 Zahra Nazemian: *Invariant Subspaces and the Characterization Problem of Affine n-spaces*

Coffee break

12:00–12:25 Joscha Prochno: *The probabilistic behavior of lacunary sums*

12:30–12:55 Matej Filip: *Laurent polynomials and deformations of non-isolated Gorenstein toric singularities*

13:00–13:25 Ana Prlić: *Unitary Highest Weight Modules with Fixed Infinitesimal Character*

14:00– Lunch at Restoran "Kod Pere", Rubetićeva 25

Abstracts

Luka Boc Thaler (University of Ljubljana)

On the exponential tower functions

We introduce a new class of entire functions $\mathcal{E}$ (exponential tower functions) and discuss its properties. In particular we will prove that $\mathcal{E}$ is a dense subset of the set of all non-vanishing entire functions $\mathcal{O}^*(\mathbb{C})$, and that for every closed set $V \subseteq \mathbb{C}$ that contains the origin and at least one more point, there exist a locally univalent function in our class $\mathcal{E}$ whose set of singular values is equal to V.

Matej Filip (University of Ljubljana)

Laurent polynomials and deformations of non-isolated Gorenstein toric singularities

We establish a correspondence between one-parameter deformations of an affine Gorenstein toric variety X, defined by a polytope P, and mutations of a Laurent polynomial f, whose Newton polytope is equal to P. If the Newton polytope P of f is two dimensional and there exists a set of mutations of f that mutate P to a smooth polygon, then, we show that the Gorenstein toric variety, defined by P, admits a smoothing. This smoothing is obtained by proving that the corresponding one-parameter deformation families are unobstructed and that the general fiber of this deformation family is smooth.

Zahra Nazemian (University of Graz)

Invariant Subspaces and the Characterization Problem of Affine n-spaces

In a Bourbaki seminar [3], Kraft identified the *Characterization Problem* for polynomial rings as one of the eight central challenges in affine algebraic geometry, alongside the Jacobian Conjecture, the Automorphism Problem, and the Zariski Cancellation Problem (ZCP).

We prove the following result (see [2]):

Theorem. *Let K be an algebraically closed field of characteristic zero, and let $A \neq K$ be a (not necessarily commutative) algebra. Then*

$$A \cong K[z_1, \ldots, z_m]$$

for some $m \geq 2$ if and only if the following two conditions hold:

1. *A is finitely generated and connected graded;*
2. *A has no nontrivial invariant subspaces.*

Condition (1) is essential: for example, the Weyl algebras satisfy condition (2), but not condition (1), and are not isomorphic to polynomial rings. It remains an open question whether condition (1) can be replaced by a weaker assumption or dropped in the commutative case. If time permits, we will discuss invariant subspaces of Grassmann algebras, which have a structure very close to that of commutative polynomial rings; see [1].

References:

1. M. K. Demir, Z. Nazemian, *Aut-stable subspaces of Grassmann algebras*, `https://arxiv.org/pdf/2508.16945`
2. H. Huang, Z. Nazemian, Y. Wang, and J. J. Zhang, *Relative cancellation*, to appear in Proc. Amer. Math. Soc., 2025, 11 pp.
3. H. Kraft, *Challenging problems on affine n-space*, in Séminaire Bourbaki, Vol. 1994/95, Astérisque **237** (1996), Exp. No. 802, 295–317, Société Mathématique de France.

Ana Prlić (University of Zagreb)

Unitary Highest Weight Modules with Fixed Infinitesimal Character

In this talk, we study unitary highest weight $(\mathfrak{g}, K)$-modules for a noncompact, connected, simply connected simple Lie group G, assuming (G, K) is a Hermitian symmetric pair. Each such module is determined by a highest weight λ, with infinitesimal character $\Lambda = \lambda + \rho$. We present a criterion

for when a parameter Λ, conjugate to its $\mathfrak{g}$-dominant representative, corresponds to a unitary module and also provide a new, more elementary proof of the classification of unitary highest weight modules originally obtained by Enright–Howe–Wallach and independently by Jakobsen.

The talk is based on joint work with Pavle Pandžić, Gordan Savin, Vladimír Souček, and Vít Tuček.

Joscha Prochno (University of Passau)

The probabilistic behavior of lacunary sums

It is known through classical works of Kac, Salem, Zygmund, Erdös and Gal that lacunary sums behave in several ways like sums of independent random variables, satisfying, for instance, a central limit theorem or a law of the iterated logarithm. We present some recent results on their large deviation behavior, which show that on this scale, contrary to the scale of the CLT or the LIL, the LDP is sensitive to the arithmetic properties of the underlying Hadamard gap sequence. If time allows, we shall briefly discuss some recent results regarding the optimality of Diophantine conditions in the law of the iterated logarithm for lacunary systems.

Mateo Tomašević (University of Zagreb)

Classification of Jordan multiplicative maps on matrix algebras

A central theme in ring theory is understanding the interplay between the multiplicative and additive structures. A classical result of Martindale shows that any bijective multiplicative map from a prime ring containing a nontrivial idempotent onto an arbitrary ring is automatically additive, and thus a ring isomorphism. Another important contribution, due to Jodeit and Lam, classifies non-degenerate multiplicative self-maps on matrix rings over principal ideal domains, showing in particular that all bijective such maps are necessarily additive.

Every associative ring or algebra naturally carries the structure of a *Jordan algebra* via the *symmetrized (Jordan) product* $x \diamond y := xy + yx$. This

structure plays a fundamental role in various areas, including the foundations of quantum mechanics. In this setting, Molnar proved that all bijective Jordan multiplicative maps between standard operator algebras are additive, provided the domain has dimension greater than one.

In this talk, we give a complete classification of Jordan multiplicative self-maps on full matrix algebras $M_n(\mathbb{F})$, where $\mathbb{F}$ is a field of characteristic different from 2. We show that a sharp dichotomy holds: such maps are either constant with an idempotent value, or additive, and hence Jordan monomorphisms. For the fields $\mathbb{R}$ or $\mathbb{C}$, we extend this classification along with the Jodeit–Lam theorem to injective Jordan multiplicative maps on *structural matrix algebras*, which are subalgebras of $M_n(\mathbb{F})$ containing all diagonal matrices. We give a precise characterization of when such maps are automatically additive and describe their structure explicitly. These results reveal strong rigidity phenomena that arise from Jordan product.

The talk is based on joint work with Ilja Gogić.