

FIGURE 6.1. In each panel 100 pairs x_i, y_i are generated at random from the blue curve with Gaussian errors: $Y = \sin(4X) + \varepsilon$, $X \sim U[0, 1]$, $\varepsilon \sim N(0, 1/3)$. In the left panel the green curve is the result of a 30-nearest-neighbor running-mean smoother. The red point is the fitted constant $\hat{f}(x_0)$, and the red circles indicate those observations contributing to the fit at x_0 . The solid yellow region indicates the weights assigned to observations. In the right panel, the green curve is the kernel-weighted average, using an Epanechnikov kernel with (half) window width $\lambda = 0.2$.

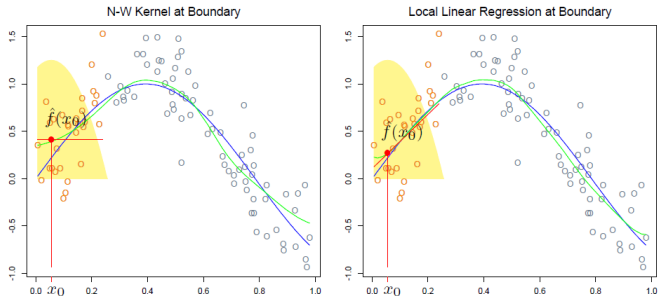


FIGURE 6.3. *The locally weighted average has bias problems at or near the boundaries of the domain. The true function is approximately linear here, but most of the observations in the neighborhood have a higher mean than the target point, so despite weighting, their mean will be biased upwards. By fitting a locally weighted linear regression (right panel), this bias is removed to first order.*

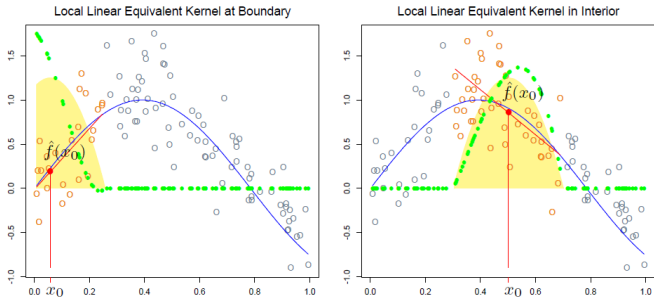


FIGURE 6.4. The green points show the equivalent kernel $l_i(x_0)$ for local regression. These are the weights in $\hat{f}(x_0) = \sum_{i=1}^N l_i(x_0)y_i$, plotted against their corresponding x_i . For display purposes, these have been rescaled, since in fact they sum to 1. Since the yellow shaded region is the (rescaled) equivalent kernel for the Nadaraya–Watson local average, we see how local regression automatically modifies the weighting kernel to correct for biases due to asymmetry in the smoothing window.

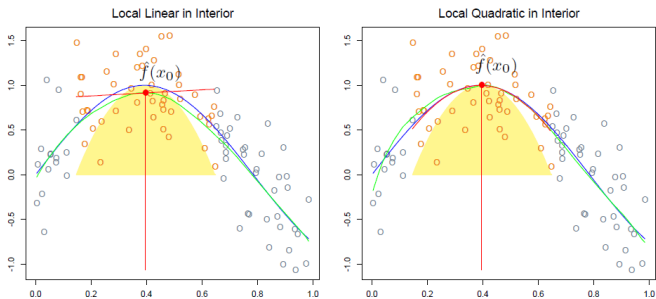


FIGURE 6.5. Local linear fits exhibit bias in regions of curvature of the true function. Local quadratic fits tend to eliminate this bias.

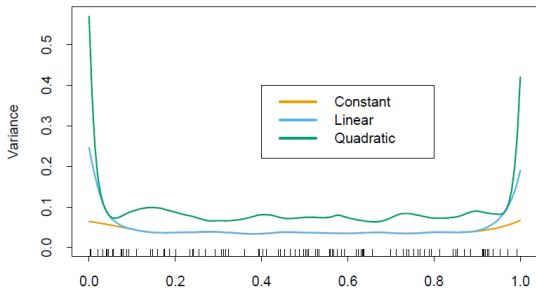


FIGURE 6.6. The variances functions $\|l(x)\|^2$ for local constant, linear and quadratic regression, for a metric bandwidth ($\lambda = 0.2$) tri-cube kernel.